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DESIGN OF LOW ORDER CONTROLLERS FOR ROBUST DISTURBANCE
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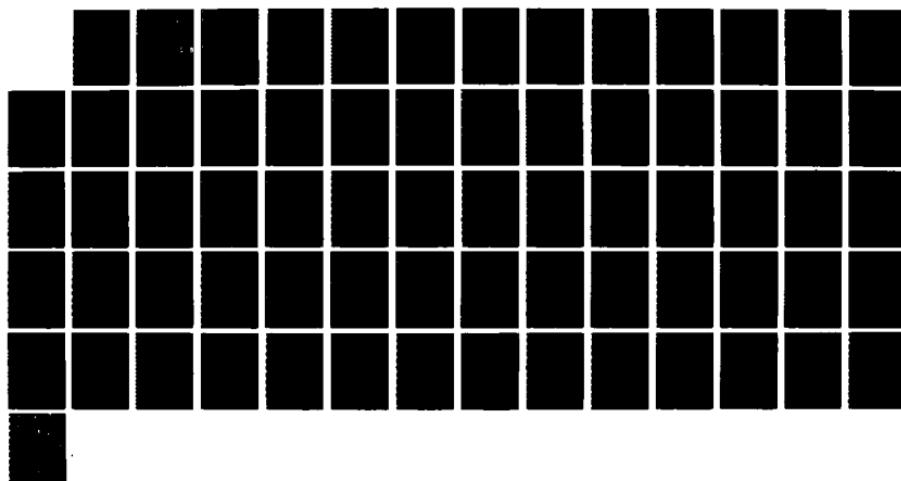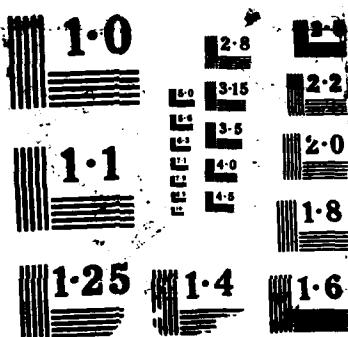
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DESIGN OF LOW ORDER CONTROLLERS FOR ROBUST DISTURBANCE REJECTION IN LARGE SPACE STRUCTURES

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Russell Allen Ramaker

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

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This thesis will investigate several issues relating to the objective of improving the disturbance rejection of a Large Space Structure (LSS) type system. Due to properties of LSS systems, the design of the controller must address three main issues: 1) improving of the disturbance rejection properties of the system, 2) insuring that the controller is robust to modeling uncertainty and 3) implementing the design as a low order output feedback controller.

The disturbance rejection of the system will be quantified using the H_∞-norm. An arbitrary level of disturbance rejection will then be achieved through a Linear Quadratic (LQ) minimization. This LQ solution produces a state feedback controller which is robust to modeling uncertainty. In order to realize this design, a low order output feedback controller will be designed based on the LQ solution using projective controls.

The method described in this thesis will then be applied to a 40th order LSS example

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Using a decentralized approach, a controller will be designed which satisfies the issues discussed above.

DESIGN OF LOW ORDER CONTROLLERS FOR ROBUST
DISTURBANCE REJECTION IN LARGE SPACE STRUCTURES

BY

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THESIS

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1. INTRODUCTION

This thesis investigates the problem of disturbance rejection for a class of problems commonly known as Large Space Structures (LSS). A solution method will be developed herein which reduces the effect of the disturbance on the system while accounting for uncertainties present in the problem. The method will employ H_∞ concepts, LQ minimization, and projective controls in achieving this goal.

Certain characteristic properties of Large Space Structures pose particularly difficult problems for system designers. Typically, the systems themselves are of infinite order and are modeled as high order systems. Because of this, models of LSS systems tend to have uncertain parameter values. The modes of the system tend also to be very lightly damped and thus easily go unstable. Also, the number of inputs and outputs, while being multiple, are small compared to the order of the system. In some cases, decentralized controllers are necessary due to physical constraints; in other cases, the structure of the system suggests that a decentralized controller is appropriate.

Because of the small number of outputs available and the large order of the system, high order observers are often necessary to implement state feedback controllers obtained by optimization methods such as LQG. However, observers of such high order are not practical and fortunately are not often necessary. Thus an important design problem becomes one of designing controllers of low order.

One approach to the low order controller design problem is to use a low order design model. This will result in a low order observer. In such a case, the choice of the design system becomes very important. Because of the low damping of the system, the effects of spillover on the dynamics not included in the design model can be very undesirable. Model reduction, however, may have to

be undertaken since design computations may be impossible for large order systems.

Another approach is to design a high order controller based on the full order model of the system and then reduce the order of the controller using open loop techniques. This also can cause stability problems since no account is taken of the system in making the reduction. If, however, the full order model is used in finding the reduced order controller, problems with spillover can be avoided at the design stage.

This is the approach that will be used in this thesis. First a full-state (high-order) controller will be designed which possesses the desired robustness and disturbance rejection properties. A low-order controller will then be designed based on the full-state solution using projective controls. The projective controls approach allows one to retain properties of the full-state solution. In addition, the parameters of the controller can be adjusted to improve the non-retained dynamics directly. This, in a sense, integrates the synthesis of the low-order controller with the evaluation of the "unmodeled" dynamics.

This approach will be applied to an example which exhibits LSS properties. Tharp (1987, Tharp, et al., 1987) recently studied this example in the context of improving the damping of the system. This was essentially a time domain approach which improved the response of the system to initial conditions. Tharp's method did, however, use a frequency domain approach to the robustness issue.

In this work, the response of the system to an external disturbance will be improved. A method of solution will be developed that takes into account both the uncertainty in the form of the disturbance and in the parameters of the system itself. The former is handled through use of an H_∞-norm as a "worst-case" analysis of the effect of a class of disturbances. The latter is handled through the robustness properties of a linear-quadratic (LQ) minimization. Projective controls are then used to retain properties of the LQ solution for an output feedback solution of the problem.

2. THE DISTURBANCE REJECTION PROBLEM

2.1. Problem Definition.

The problem of disturbance rejection is a common one for system designers. It arises when the system to be regulated possesses an external input which disturbs the systems from its desired state. In order to reduce the effect of this disturbance input on the system's output, control can be applied. Since the disturbance cannot be measured directly, an output feedback controller can be useful in reducing the sensitivity of the system to the disturbance provided the controller also stabilizes the system. This, essentially, is the disturbance rejection problem. Note that the term "disturbance rejection" is used to mean reduced sensitivity to disturbances, not the complete elimination of the effect of the disturbance as other methods seek to accomplish.
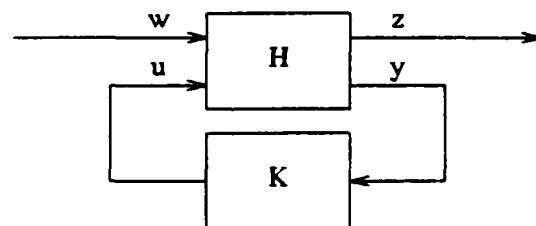
Figure 2.1. The Disturbance Rejection Problem.

Fig. 2.1 illustrates the general problem where $w(t)$ is the disturbance input, $u(t)$ is the controlled input, $z(t)$ is the controlled output and $y(t)$ is the measured output. The goal is to choose the feedback, $K:y \to u$ so that the system response to the disturbance, $H:w \to z$ is sufficiently small in some sense. This can be seen as a regulator problem with z being the regulated output.

In order to define what is meant by a small response to a disturbance, a measure of the size of the response will be defined. As noted by Chang and Pearson (1984), the type of norm used is

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dependent upon what is known about the disturbance. If the form of the disturbance is known precisely, either deterministically or stochastically, then a quadratic norm can be applied and an LQG solution obtained. However, if little is known about the disturbance, i.e., only the class of signals to which the disturbance belongs is known, then a so-called worst-case measure of the disturbance may be more appropriate. Using such a measure, a system can be designed so that the worst possible disturbance in a given class of disturbances has a satisfactory response.

The "worst-case" effect of a disturbance belonging to a class of signals, C on a system can be specified using the following definitions.

Definition 2.1. *Let* $x:[0,\infty) \to \mathbb{R}^n$ *be a square-integrable function. Then the* L_2*-norm of* x *is*

$$\|x\|_2 \overset{\Delta}{=} \left[\int_0^\infty x(t)^T x(t) \, dt \right]^{1/2}. \tag{2.1}$$

Definition 2.2. *Given a system,* $H:w \to z$ *and a class of signals,* C, *define the* **gain of** H **with respect to** C *as*

$$g(H,C) \overset{\Delta}{=} \sup \left\{ \frac{\|z\|_2}{\|w\|_2} : w \in C, \ w \neq 0 \right\} \tag{2.2}$$

where w *is square-integrable.*

Consider a class of signals defined by the L_2-norm of the signal, i.e.,

$$S_\rho \overset{\Delta}{=} \{ x : \|x\|_2 < \rho \}. \tag{2.3}$$

Note that the L_2-norm of a signal is equivalent to the "energy" of the signal. Thus, the class S_ρ consists of all signals with "energy" less than a prescribed level ρ. Since this may be all that is known about a signal, this can be a very useful class of signals.

These definitions will now be used to quantify the amount of disturbance rejection possessed by a system.

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Definition 2.3. *Given a system,* $H:w \to z$ *and a class of disturbances,* S_ρ, *then* H *has disturbance rejection* γ *provided H is stable and*

$$g(H.S_\rho) \leqslant \gamma. \tag{2.4}$$

Consider a system with disturbance rejection γ. Then for any disturbance $w(t)$ in the class S_ρ, the resulting signal at the output $z(t)$ belongs to the class $S_{\rho\gamma}$, i.e., if $\|w(t)\|_2 < \rho$ then $\|z(t)\|_2 < \rho\gamma$.

2.2. Linear Systems Case.

If only finite-dimensional linear time-invariant (FDLTI) systems and controllers are considered, then the system, H and the controller, K can be expressed in terms of rational transfer functions $H(s)$ and $K(s)$ respectively, i.e.,

$$z(s) = H(s)w(s) \tag{2.5a}$$

$$u(s) = K(s)y(s) \tag{2.5b}$$

where $u \in \mathbb{R}^m$ is the controlled input, $w \in \mathbb{R}^q$ is the disturbance input, $y \in \mathbb{R}^r$ is the measured output, and $z \in \mathbb{R}^s$ is the controlled output.

FDLTI systems can also be expressed in state space. For the system H, a minimal order state space representation of the system is of the form

$$\dot{x} = Ax + Bu + Dw \tag{2.6a}$$

$$y = Cx \tag{2.6b}$$

$$z = Ex \tag{2.6c}$$

where $x \in \mathbb{R}^n$ is the state vector. A minimal order state space representation of the controller is

$$\dot{v} = H_o x + D_o u \tag{2.7a}$$

$$u = -N_d v - K_d y \tag{2.7b}$$

where $v \in \mathbb{R}^p$ is the state vector. The connection between the two representations is given by the following expressions.

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix} (sI-A)^{-1} [D \ B] \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \qquad (2.8a)$$

$$u(s) = -[N_d(sI-H_o)^{-1}D_o + K_d]y(s) . \qquad (2.8b)$$

For linear systems, the gain of the system is independent of the "energy" of the disturbance, p and can be expressed in terms of the H_∞-norm of the system.

Definition 2.4. *Let* H(s) *be a stable, proper, MIMO transfer function. Then the* H_∞*-norm of* H(s) *is*

$$\|H(s)\|_\infty \overset{\Delta}{=} \sup_\omega \ \bar{\sigma}(H(j\omega)) \qquad (2.9)$$

where $\bar{\sigma}$ *denotes the maximum singular value.*

Theorem 2.1. *Let* H(s) *be a stable, proper, MIMO transfer function. Then* H(s) *has disturbance rejection* γ *iff*

$$\|H(s)\|_\infty \leqslant \gamma . \qquad (2.10)$$

Proof : By Theorem 2, Chapter 2 (Francis, 1986), $g(H.S_\rho) = \|H(s)\|_\infty$. Thus, (2.3) and (2.10) are equivalent for linear systems. $\square$

2.3. Solutions of the Disturbance Rejection Problem.

A number of methods are available to solve the disturbance rejection problem. One method is disturbance decoupling (Wonham, 1979). This method uses geometric concepts to decouple the output from the disturbance (i.e., achieve $\gamma \equiv 0$) and results in a state feedback controller. However, solutions to this problem exist only under relatively severe conditions usually not satisfied in many systems.

When the above existence conditions cannot be met, the problem can be approached as an H_∞ minimization problem (Zames, 1979). (An overview of solution techniques can be found in Safonov, et al., 1987.) This method produces a stabilizing controller which minimizes γ. The

resulting controller is a dynamic output feedback of relatively high order, generally higher than the order of the system. Peterson (1987) has developed a method which uses the ARE to asymptotically achieve the H_∞ minimum for a state feedback controller.

It may not always be necessary to achieve the minimum γ. In other cases it may not be possible to apply the control authority necessary to implement the above controllers. Also, robustness issues must be addressed in order to deal with uncertain systems such as LSS systems. For these reasons, LQ based solutions that bound the disturbance rejection are attractive. One such method is given by Imai, et al. (1986) based on a frequency weighted LQ minimization. The state weightings are chosen to frequency-shape the return difference matrix. This can then be used to solve the disturbance rejection problem. However, the method requires $R\{D\} \subseteq R\{B\}$ where $R\{\cdot\}$ denotes range space. Furthermore, the method produces a dynamic state feedback controller as the solution to the disturbance rejection problem.

The method proposed in the next section is related to the results due to Imai in the sense that it also requires $R\{D\} \subseteq R\{B\}$ and yields a state feedback, but does not resort to the frequency-weighted LQ approach. Therefore, since the controller type is the same, methods used to implement the resulting state feedback are valid for the method of Imai as well.

2.4. LQ Disturbance Rejection.

In this section, the properties of the LQ method will be used to establish disturbance rejection properties. In addition, since the solution is the result of an LQ minimization, certain robustness properties are guaranteed. As shown in Lehtomaki, et al. (1981), minimum gain margins are $1/2, \infty$ and minimum phase margins are ± 60 degrees.

Theorem 2.2. *Given a system* (2.6) *such that* $R\{D\} \subseteq R\{B\}$, *let* $\alpha \in \mathbb{R}^{m \times p}$ *where* $D = B\alpha$. *Let* $u = -Kx$ *where*

$$K = \gamma^{-2} \alpha D^T P \qquad (2.11)$$

and P be such that

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$$A^T P + PA + E^T E - \gamma^{-2} PDD^T P = 0 . \qquad (2.12)$$

then the system has disturbance rejection γ.

Proof: Since $R\{D\} \subseteq R\{B\}$. $D = B\alpha$. Let

$$K = R^{-1} B^T P \qquad (2.13)$$

be a state feedback gain matrix where P solves the ARE

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \qquad (2.14)$$

with $Q \geqslant 0$ and $R > 0$. Let $F = A - BK$ and $Q = E^T E$. Then (2.14) becomes

$$F^T P + PF + E^T E + PBR^{-1} B^T P = 0 . \qquad (2.15)$$

Then by Willems (1971), Lemma 1,

$$[E(j\omega I - F)^{-1} B]^* [E(j\omega I - F)^{-1} B] \leqslant R \qquad (2.16)$$

where * denotes complex conjugate transpose. Premultiplying (2.16) by α^T and postmultiplying by α yields

$$[E(j\omega I - F)^{-1} D]^* [E(j\omega I - F)^{-1} D] \leqslant \alpha^T R \alpha . \qquad (2.17)$$

Thus, if R is chosen such that

$$\alpha^T R \alpha \leqslant \gamma^2 I_p . \qquad (2.18)$$

then from (2.17)

$$\lambda_{max}\{[H(j\omega)]^* [H(j\omega)]\} \leqslant \lambda_{max}\{\gamma^2 I_p\} = \gamma^2 . \qquad (2.19)$$

Thus, since $H(s) = E(sI - F)^{-1} D$,

$$\|H(s)\|_\infty \overset{\Delta}{=} \bar{\sigma}[E(j\omega I - F)^{-1} D] \leqslant \gamma \qquad (2.20)$$

and the system has disturbance rejection γ.

If $m = p$, let $R = \gamma^2 \alpha^{-T} \alpha^{-1}$. Substituting R and Q into (2.14) gives (2.12). However, if $m > p$, any R that satisfies (2.18) can be chosen. Introduce a transformation of inputs, $u = T\tilde{u}$ where $T \in \mathbb{R}^{m \times m}$. Then,

$$\tilde{B} = BT \qquad (2.21)$$

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$$\tilde{R} = T^T R T \tag{2.22}$$

and (2.14) becomes

$$A^T P + P A + Q - P\tilde{B}\tilde{R}^{-1}\tilde{B}^T P = 0 . \tag{2.23}$$

Let $T = [\alpha \ \gamma]$ where γ is defined so that T is nonsingular. Then

$$\tilde{B} = [D \ B\gamma], \quad \tilde{R} = \begin{vmatrix} \alpha^T R\alpha & \alpha^T R\gamma \\ \gamma^T R\alpha & \gamma^T R\gamma \end{vmatrix} . \tag{2.24}$$

Let $\alpha^T R\alpha = \gamma^2 I_p$. Then, as $\gamma^T R\gamma \to \infty$

$$\tilde{R}^{-1} \to \begin{vmatrix} \gamma^{-2}I_p & 0 \\ 0 & 0 \end{vmatrix} . \tag{2.25}$$

Substituting (2.24) and (2.25) into (2.23) and (2.13) yields (2.12) and (2.11) respectively proving

the result. $\square$

Thus, by the methods of this chapter, a system's disturbance rejection properties can be

characterized by a "worst-case" measure, the H_∞-norm. Furthermore, an arbitrary level of distur-

bance rejection can be achieved through an LQ minimization which also provides good robustness

characteristics. In the coming chapters, methods will be developed which use projective controls to

implement the state feedback controller with low order output feedback controllers.

3. PROJECTIVE CONTROLS

3.1. Problem Description.

When designing controllers with optimization methods such as LQ minimization, a state feedback results which achieves the optimum. However, in order to implement this design, an output controller must be used. The state feedback system can be viewed as a reference system which possesses properties to be retained by the output feedback system. Often this is done is by designing a full-order observer and then, if necessary, reducing the controller order by either directly approximating the controller with a lower order controller, or by reducing the system order before the observer is designed.

Projective controls, as developed by Medanic (1978, 1979), offers a method of implementing an output feedback controller which retains a subset of the reference system's eigenvalues and eigenvectors. The controller can be either static or dynamic. In the dynamic controller case the order is determined by the designer to meet design objectives. The retained eigenstructure is arbitrary and can be chosen to retain certain properties of the reference system deemed to be important. In addition, it will be seen that design freedoms in the dynamic controller cases can be used to shape the dynamics of the residual (non-retained) dynamics. In this chapter, basic features of the projective controls approach will be reviewed. These are essentially time domain properties. In Chapter 4, frequency domain properties of systems with projective controllers will be developed and applied to the disturbance rejection problem.

The basic problem is the following. Given a system

$$\dot{x} = Ax + Bu \tag{3.1a}$$

$$y = Cx. \tag{3.1b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^r$ and a state feedback control.

$$u = -Kx \tag{3.2}$$

which defines the desired characteristics of the closed loop system. the resulting reference dynamics are

$$\dot{x} = Fx. \quad F \stackrel{\Delta}{=} A - BK \tag{3.3}$$

It's eigenstructure is characterized by

$$FX = X\Lambda \tag{3.4}$$

where F is the reference system matrix. X is the eigenvector matrix and Λ is a diagonal matrix of the eigenvalues of F. $\lambda(F)$. The goal of projective controls is to design a feedback using only the outputs of the system which retains a subset of the reference dynamics. The following is a review of results found in Hopkins. et al. (1981) and Medanic. et al. (1985). Discrete-time results are given in Medanic and Uskokovic (1983).

3.2. Static Projective Controls.

The static projective controller is determined by applying a projection of the state feedback matrix. K into the space spanned by the output matrix. C such that a r-dimensional subspectrum $\lambda_r \subseteq \lambda(F)$ is retained. If $P = X_r(CX_r)^{-1}C$ is the projection matrix where X_r is the matrix of eigenvectors associated with λ_r. then KP is the desired projection. The resulting closed loop system matrix is

$$A_c = A - BKP. \tag{3.5}$$

As a result of the projection. the control can be implemented in terms of available outputs. i.e..

$$u = -K_s y \tag{3.6}$$

where

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$$K_s = KX_r(CX_r)^{-1}. \tag{3.7}$$

In order to determine the spectrum of A_c, define a transformation $T \in \mathbb{R}^{n \times n}$ where

$$T = [X_r \ \ Y], \qquad T^{-1} = \begin{bmatrix} U \\ V \end{bmatrix} \tag{3.8}$$

and $CY = 0$. The transformed closed loop system matrix is

$$\tilde{A}_c = T^{-1}A_cT = \begin{bmatrix} \Lambda_r & UAY \\ 0 & VAY \end{bmatrix}. \tag{3.9}$$

Thus, the spectrum of the projective system is

$$\lambda(A_c) = \lambda_r \cup \lambda(A_r) \tag{3.10}$$

$$A_r \overset{\Delta}{=} VAY. \tag{3.11}$$

Therefore, the dynamics of the closed-loop system consist of a subset of the reference dynamics, λ_r, and the residual dynamics, $\lambda(A_r)$.

Explicit expressions for U, V, and Y can be derived by assuming that the output structure is of the form $C = [I, 0]$. Subdividing the matrices compatible with C yields

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}, \quad X_r = \begin{bmatrix} X_{r1} \\ X_{r2} \end{bmatrix}. \tag{3.12}$$

Thus, the parameters of T can be found to be

$$Y = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad U = [X_{r1}^{-1} \ \ 0], \quad V = [-N_0 \ \ I] \tag{3.13}$$

where,

$$N_0 \overset{\Delta}{=} X_{r2}X_{r1}^{-1}. \tag{3.14}$$

This gives us explicit expressions for A_r and K_s:

$$A_r = A_{22} - N_0A_{12} \tag{3.15}$$

$$K_s = K_1 + K_2N_0. \tag{3.16}$$

One should note that there is no guarantee of stability in this method as with any static controller. The residual dynamics determine the stability of the system. However, if the number of

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outputs is sufficient to capture the desired properties and the residual dynamics are satisfactory, then a static projective controller may be sufficient to meet the design goals of the system.

3.3. Dynamic Projective Controls.

If a static controller is not sufficient to meet the design goals, a dynamic projective controller may be necessary. Consider a p-dimensional controller with a state space representation of the form

$$\dot{z} = Hz + Dy \tag{3.17a}$$

$$u = -K_d y - N_d z . \tag{3.17b}$$

where $z \in \mathbb{R}^p$ is the state vector. Appending the dynamics of the system (3.1) to the dynamics of the controller (3.17), an extended system can be defined:

$$\dot{x}_e = A_e x_e + B_e u \tag{3.18a}$$

$$y_e = C_e x_e \tag{3.18b}$$

where,

$$
x_e \overset{\Delta}{=} \begin{bmatrix} z \\ x \end{bmatrix}, \quad y_e \overset{\Delta}{=} \begin{bmatrix} z \\ y \end{bmatrix}, \quad A_e = \begin{bmatrix} H & DC \\ 0 & A \end{bmatrix}, \quad B_e = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad C_e = \begin{bmatrix} I_p & 0 \\ 0 & C \end{bmatrix}. \tag{3.19}
$$

The reference system matrix in the extended system is $F_e = A_e - B_e K_e$ where

$$
K_e = \begin{bmatrix} 0 & K \end{bmatrix}, \quad F_e = \begin{bmatrix} H & DC \\ 0 & F \end{bmatrix}. \tag{3.20}
$$

Since there are r outputs from the system and p outputs from the compensator, an r-dimensional subspectrum $\lambda_r \subseteq \lambda(F)$ and a p-dimensional subspectrum $\lambda_p \subseteq \lambda(F)$ can be retained. In the extended system, this can be expressed

$$F_e X_{re} = X_{re} \Lambda_{re} \tag{3.21}$$

where,

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$$X_{re} = \begin{bmatrix} W_r & W_p \\ X_r & X_p \end{bmatrix}, \quad \Lambda_{re} = \begin{bmatrix} \Lambda_r & 0 \\ 0 & \Lambda_p \end{bmatrix}. \tag{3.22}$$

In the extended system, the controller can be represented as $u = -K_{se}y_e$ where

$$K_{se} = \begin{bmatrix} N_d & K_d \end{bmatrix}. \tag{3.23}$$

The form of this extended system is identical to the static case. Thus, the previous results for static projective controllers (3.7) and (3.11) can be applied, i.e.,

$$K_{se} = K_e X_{re}(C_e X_{re})^{-1} \tag{3.24}$$

$$A_{re} = V_e A_e Y_e. \tag{3.25}$$

Assuming $C = [I_r \ 0]$ and subdividing as in (3.12), it can be shown that

$$X_{re} = \begin{bmatrix} W_r & W_p \\ X_{r1} & X_{p1} \\ X_{r2} & X_{p2} \end{bmatrix}, \quad Y_e = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \tag{3.26}$$

$$U_e = \begin{bmatrix} W_p^{-1} & -(I + P_o X_{p1})L X_{r1}^{-1} & 0 \\ -(X_{r1} - X_{p1}L)^{-1} X_{p1} W_p & (X_{r1} - X_{p1}L)^{-1} & 0 \end{bmatrix}, \quad V_e = \begin{bmatrix} -B_o W_p^{-1} & (B_o P_o - N_o) & I \end{bmatrix} \tag{3.27}$$

where

$$B_o \overset{\Delta}{=} X_{p2} - N_o X_{p1} \tag{3.28}$$

$$L \overset{\Delta}{=} W_p^{-1} W_r \tag{3.29}$$

$$P_o \overset{\Delta}{=} L(X_{r1} - X_{p1}L)^{-1}. \tag{3.30}$$

The closed-loop spectrum of the extended system is thus

$$\lambda(A_{ce}) = \lambda_r \cup \lambda_p \cup \lambda(A_{re}) \tag{3.31}$$

where,

$$A_{re} = A_r + B_o P_o A_{12}. \tag{3.32}$$

W_p can be seen to represent a similarity transformation of the controller parameters (Medanic, et al., 1985). Letting $W_p = (I + P_o X_{p1})$, the controller parameters are then

$$H = \Lambda_p + P_o F_{12} B_o \tag{3.33a}$$

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$$D = P_oF_r - HP_o \tag{3.33b}$$

$$K_d = K_s - K_2B_oP_o \tag{3.33c}$$

$$N_d = K_2B_o \tag{3.33d}$$

where $F_{12} = A_{12} - B_1K_2$ and $F_r = A_{11} - B_1K_1 + F_{12}N_o$. Since W_r is arbitrary, $P_o \in \mathbb{R}^{p \times r}$ parameterizes all controllers of the form (3.17) which retain the modes associated with λ_p and λ_r.

3.4. Strictly Proper Dynamic Controller

In many cases it may be desirable to design a projective controller that is strictly proper. Since controllers of this type rolloff at high frequencies, such a controller has improved robustness properties at high frequencies. A strictly proper controller is obtained if only compensator states are used in the feedback. The following is a summary of results presented by Tharp (1987).

Consider a p-dimensional strictly proper controller with a state space representation of the form

$$\dot{z} = H_oz + D_oy \tag{3.34a}$$

$$u = -K_oz \tag{3.34b}$$

where $z \in \mathbb{R}^p$ is the state vector. The extended system is given by (3.18) where

$$x_e = \begin{bmatrix} z \\ x \end{bmatrix}, \quad y_e = z, \quad A_e = \begin{bmatrix} H_o & D_oC \\ 0 & A \end{bmatrix}, \quad B_e = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad C_e = \begin{bmatrix} I_p & 0 \end{bmatrix}. \tag{3.35}$$

The reference dynamics are given by $F_e = A_e - B_eK_e$ where

$$K_e = \begin{bmatrix} 0 & K \end{bmatrix}, \quad F_e = \begin{bmatrix} H_o & D_oC \\ 0 & F \end{bmatrix}. \tag{3.36}$$

Since the extended system now has p outputs, only a p-dimensional subspectrum, λ_p of the reference dynamics, $\lambda(F)$ can be retained. Thus,

$$F_eX_{re} = X_{re}\Lambda_{re} \tag{3.37}$$

where,

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$$X_{re} = \begin{bmatrix} W_p \\ X_p \end{bmatrix}, \quad \Lambda_{re} = \Lambda_p.$$

(3.38)

The controller (3.34) can be represented as $u = -K_{se} y_e$ where

$$K_{se} = K_o.$$

(3.39)

Applying (3.7) and (3.11) as before yields

$$K_{se} = K_e X_{re} (C_e X_{re})^{-1}$$

(3.40)

$$A_{re} = V_e A_e Y_e.$$

(3.41)

Let T be a transformation as in (3.8). Then

$$Y_e = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad U_e = \begin{bmatrix} W_p^{-1} & 0 \end{bmatrix}, \quad V_e = \begin{bmatrix} X_p W_p^{-1} & I \end{bmatrix}.$$

(3.42)

The spectrum of the closed-loop system is then

$$\lambda(A_{ce}) = \lambda_p \cup \lambda(A_{re})$$

(3.43)

where

$$A_{re} = A - X_p D_o C.$$

(3.44)

Letting $W_p = I$, the controller parameters are

$$H_o = \Lambda_p - D_o C X_p$$

(3.45a)

$$K_o = K X_p$$

(3.45b)

where $D_o \in \mathbb{R}^{p \times r}$ is a free parameter. Thus, D_o parameterizes all controllers of the form (3.34) which retain the modes associated with λ_p.

The methods of this chapter present a method by which low order output feedback controllers can be designed which retain modes of a state feedback system. Results are presented for three controller configurations: static controllers, proper dynamic controllers, and strictly proper dynamic controllers. In the case of static controllers, the selection of modes to be retained completely specifies the controller. However, in both dynamic controller cases, a parameterization of all controllers which retain the specified modes is obtained. Since this design freedom affects the

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residual dynamics, the overall system performance can be affected by the chosen parameters. Thus, in the next chapter, frequency properties of projective controllers will be studied for application to the disturbance rejection problem.

4. FREQUENCY PROPERTIES OF PROJECTIVE CONTROLLERS

4.1. Frequency Properties in the Generalized Case.

The previous chapter details a method for retaining properties of a state feedback with an output feedback through use of a projective controller. Since the system's disturbance rejection properties are based on a frequency domain measure, i.e., the H_∞ norm, it would be useful to have an expression relating the frequency domain properties of a system with a state feedback controller to the frequency domain properties of a system with a projective controller.

Given a system with a disturbance (2.6) and a reference state feedback control $u=-Kx$, the response of the system $z_r(s)$ to the disturbance $w(s)$ is

$$z_r(s) = E(sI - F)^{-1}Dw(s) \overset{\Delta}{=} W(s)w(s) \qquad (4.1a)$$

where $F=A-BK$. If the system has a projective controller which retains λ_r, then $u=-K_s y$ where K_s is given by (3.7). The corresponding response $z_p(s)$ is

$$z_p(s) = E(sI - A_c)^{-1}Dw(s) \overset{\Delta}{=} \overline{W}(s)w(s) \qquad (4.1b)$$

where $A_c=A-BK_sC$.

Theorem 4.1. *Let* $W(s)$ *and* $\overline{W}(s)$ *be defined above. Then* $\overline{W}(s) = W(s)+\tilde{W}(s)M(s)$ *where*

$$M(s) = KY(sI-VAY)^{-1}VD \qquad (4.2)$$

$$\tilde{W}(s) = E(sI-F)^{-1}B . \qquad (4.3)$$

Before proving this theorem, it will be useful to establish the following lemma.

Lemma 4.1. *Let* $R(s)=I+FY(sI-VFY)^{-1}V$ *where* $F=A-BK$ *and let* $P(s)=I+AY(sI-VAY)^{-1}V$. *Then* $P(s)=R(s)(I+BKY(sI-VAY)^{-1}V)$.

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Proof: Since F=A−BK,

$$P(s) = I + BKY(sI-VAY)^{-1}V + FY(sI-VAY)^{-1}V . \qquad (4.4)$$

Applying the Matrix Inversion Lemma to the third term of the above expression yields

$$FY(sI-VAY)^{-1}V = FY(sI-VFY)^{-1}V[I+(I-BKY(sI-VFY)^{-1}V)^{-1}BKY(sI-VFY)^{-1}V] . \qquad (4.5)$$

Using a form of the Matrix Inversion Lemma, $(I-M)^{-1}M=(I-M)^{-1}-I$ one obtains

$$(I-BKY(sI-VFY)^{-1}V)^{-1}BKY(sI-VFY)^{-1}V = (I - BKY(sI-VFY)^{-1}V)^{-1}-I . \qquad (4.6)$$

Also by the Matrix Inversion Lemma, $I+C(sI-A)^{-1}B = [I-C(sI-A+BC)^{-1}B]^{-1}$. Thus,

$$(I - BKY(sI-VFY)^{-1}V)^{-1}-I = BKY(sI-VAY)^{-1}V . \qquad (4.7)$$

Then,

$$P(s) = I + BKY(sI-VAY)^{-1}V + FY(sI-VFY)^{-1}V + FY(sI-VFY)^{-1}VBKY(sI-VAY)^{-1}V . \qquad (4.8)$$

Factoring yields

$$P(s) = [I+FY(sI-VFY)^{-1}V][I+BKY(sI-VAY)^{-1}V] . \qquad (4.9)$$

Thus, $P(s)=R(s)(I+BKY(sI-VAY)^{-1}V)$ proving the lemma. □

Using the results of Lemma 4.1, Theorem 4.1 will now be proven.

Proof: Transforming W(s) and $\overline{W}(s)$ by T, yields

$$W(s) = ET(sI-T^{-1}FT)^{-1}T^{-1}D \qquad (4.10a)$$

$$\overline{W}(s) = ET(sI-T^{-1}A_cT)^{-1}T^{-1}D . \qquad (4.10b)$$

Let T be the transformation given in (3.8). Then

$$W(s) = EX_r(sI-\lambda_r)^{-1}U[I+FY(sI-VFY)^{-1}V]D \qquad (4.11a)$$

$$\overline{W}(s) = EX_r(sI-\lambda_r)^{-1}U[I+AY(sI-VAY)^{-1}V]D . \qquad (4.11b)$$

By definition of R(s) and P(s),

$$W(s) = EX_r(sI-\lambda_r)^{-1}UR(s)D \qquad (4.12a)$$

$$\overline{W}(s) = EX_r(sI-\lambda_r)^{-1}UP(s)D . \qquad (4.12b)$$

Then by Lemma 4.1,

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$$\overline{W}(s) = EX_r(sI-\lambda_r)^{-1}UR(s)[I+BKY(sI-VAY)^{-1}V]D . \tag{4.13}$$

Thus, $\overline{W}(s) = W(s)+\tilde{W}(s)M(s)$ where

$$\tilde{W}(s) = EX_r(sI-\lambda_r)^{-1}UR(s)B . \tag{4.14}$$

Transforming by T^{-1} yields (4.3). □

The error caused by use of the projective controller can now be characterized in frequency domain as

$$E(s) \overset{\Delta}{=} \overline{W}(s) - W(s) = \tilde{W}(s)M(s) . \tag{4.15}$$

This expression can be used as both an analysis tool and as a synthesis tool for designing projective controllers. Note that $\tilde{W}(s)$ is independent of the choice of projective controller.

4.2. Static Controller Case.

For the case of static projective controllers of Section 3.2, an explicit expression for $M(s)$ can be obtained by assuming $C=[I_r \ 0]$. Recall from (3.13) that

$$Y = \begin{vmatrix} 0 \\ I \end{vmatrix} , \quad V = [-N_0 \ I] . \tag{4.16}$$

Thus, (4.2) becomes

$$M(s) = K_2(sI-A_r)^{-1}D_r \tag{4.17}$$

where

$$D_r \overset{\Delta}{=} D_2-N_0D_1 . \tag{4.18}$$

Note that $M(s)$ depends only on the choice of λ_r. Thus it has use as an analysis method for choosing λ_r.

4.3. Dynamic Controller Case.

In the case of dynamic projective controllers of Section 3.3, the above results for static projective controllers can be applied by considering the extended system of (3.18). Recall from (3.26-3.27) that

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$$Y_e = \begin{vmatrix} 0 \\ 0 \\ I \end{vmatrix} \,. \quad V_e = [-B_o \;\; (B_o P_o - N_o) \;\; I] \,. \tag{4.19}$$

Thus, (4.2) becomes

$$M(s) = K_2 (sI - A_r - B_o P_o A_{12})^{-1} (D_r + B_o P_o D_1) \,. \tag{4.20}$$

Since $E_e = [0 \;\; E]$ and B_e and F_e are given by (3.19) and (3.21) respectively, then

$$\tilde{W}(s) = E_e (sI - F_e)^{-1} B_e = E(sI - F)^{-1} B \,. \tag{4.21}$$

Thus, the effect of P_o on $E(s)$ is given by $M(s)$. $\tilde{W}(s)$ is simply a weighting matrix dependent only on the reference system. This allows P_o to be chosen so that $E(s)$ is minimized in some sense.

4.4. Strictly Proper Controller Case.

For the case of strictly proper projective controllers of Section 3.4, from the extended system (3.35) recall

$$Y_e = \begin{vmatrix} 0 \\ I \end{vmatrix} \,. \quad V_e = [X_p W_p^{-1} \;\; I] \,. \tag{4.22}$$

Thus,

$$M(s) = K(sI - A + X_p D_o C)^{-1} D \,. \tag{4.23}$$

Likewise $E_e = [0 \;\; E]$ and B_e and F_e are given by (3.35) and (3.37). Thus

$$\tilde{W}(s) = E_e (sI - F_e)^{-1} B_e = E(sI - F)^{-1} B \,. \tag{4.24}$$

The effect of the choice of the projective controller is completely specified by $M(s)$. D_o can thus be chosen in order to minimize $M(s)$.

4.5. Minimizing the Error Function.

An H_∞ approach to the problem of selecting P_o to minimize the error function is to solve

$$\mu = \min_{P_o} \| E(s) \|_\infty \tag{4.25}$$

where P_o is a constant $p \times r$ matrix. This would imply that

$$\| W(j\omega) - \overline{W}(j\omega) \| \leq \mu \qquad \forall \omega \in \mathbb{R} \qquad (4.26)$$

A similar approach can be taken in the strictly proper dynamic controller case to select D_o. Unfortunately, solutions to such minimization problems are unknown at present (Anderson and Lin, 1987). Thus, this problem formulation will not be pursued any further.

4.6. State Space Representation of M(s).

In this section the problem of minimizing $M(s)$ is cast into the state space domain. This is done in order to apply methods which rely on state space representation such as LQ optimization and projective controls.

Recall that for the case of dynamic controllers,

$$M(s) = K_2(sI - A_r - B_o P_o A_{12})^{-1}(D_r + B_o P_o D_1) . \qquad (4.27)$$

Defining a system input, $w_r \in \mathbb{R}$ and a system output, $z_r \in \mathbb{R}$ such that $z_r(s) = M(s)w_r(s)$, then $M(s)$ has the state space representation

$$\dot{x}_r = (A_r + B_o P_o A_{12})x_r + (D_r + B_o P_o D_1)w_r \qquad (4.28a)$$

$$z_r = K_2 x_r \qquad (4.28b)$$

where $x_r \in \mathbb{R}^{(n-r)}$ is the state vector. Define $u_r \in \mathbb{R}^p$ to be an input and $y_r \in \mathbb{R}^r$ to be an output such that

$$y_r \stackrel{\Delta}{=} A_{12}x_r + D_1 w_r \qquad (4.29a)$$

$$u_r \stackrel{\Delta}{=} P_o y_r . \qquad (4.29b)$$

Then (4.28) becomes

$$\dot{x}_r = A_r x_r + B_o u_r + D_r w_r \qquad (4.30a)$$

$$z_r = K_2 x_r . \qquad (4.30b)$$

This has the form of a disturbance rejection problem where $w_r \in \mathbb{R}$ is the disturbance input and $z_r \in \mathbb{R}$ is the output to be minimized. From (4.29b), P_o is seen to be the static gain from the outputs, $y_r \in \mathbb{R}^r$ to the inputs, $u_r \in \mathbb{R}^p$. Note, however, that the controller is strictly limited to r outputs with which to implement the controller. Also, y_r contains direct feedthrough of the disturbance

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which makes it unsuitable for applying projective controls.

These difficulties can be avoided by noting that for single input systems. $M(s)$ is a scalar transfer function. Thus. $M(s)=M(s)^T$ and an equivalent state space representation is

$$\dot{\bar{x}}_r = (A_r^T + A_{12}^T P_o^T B_o^T)\bar{x}_r + K_2^T w_r \qquad (4.31a)$$

$$z_r = D_r^T + D_1^T P_o^T B_o^T \bar{x}_r \qquad (4.31b)$$

where $\bar{x}_r \in \mathbb{R}^{(n-r)}$ is the state vector. Let $\bar{u}_r \in \mathbb{R}^r$ be an input and $\bar{y}_r \in \mathbb{R}^p$ be an output such that

$$\bar{y}_r \stackrel{\Delta}{=} B_o^T \bar{x}_r \qquad (4.32a)$$

$$\bar{u}_r \stackrel{\Delta}{=} P_o^T \bar{y}_r . \qquad (4.32b)$$

then a state space representation of $M(s)$ is

$$\dot{\bar{x}}_r = A_r^T x_r + A_{12}^T \bar{u}_r + K_2^T w_r \qquad (4.33a)$$

$$z_r = D_r^T \bar{x}_r + D_1^T \bar{u}_r . \qquad (4.33b)$$

This is now a disturbance rejection problem where the measurement. $\bar{y}_r \in \mathbb{R}^p$ is a simply a linear combination of the states of the system. Also. as the dimension of the compensator is increased. the number of outputs available for feedback increases. Thus. a $p=(n-r)$ dimension compensator is able to implement a full state solution to this disturbance rejection problem. Note. however. that z_r now contains direct feedthrough of the control $\bar{u}_r$. Thus. the methods of Chapter 2 can not be directly applied. This problem will be addressed in the next chapter.

4.7. Selection of the Free Parameter P_o.

In the representation of $M(s)$ given by (4.33). the problem of choosing P_o is cast as one of solving a disturbance rejection problem for a static output feedback with p outputs. In order to choose P_o. an 1 Q minimization problem will be solved. The performance criteria is chosen to be

$$J = \int_0^\infty \{z_r^T z_r + \bar{u}_r^T R \bar{u}_r\} dt \qquad (4.34)$$

where $R > 0$. The solution of this LQ problem yields a state feedback $\bar{u}_r = -K_r \bar{x}_r$. If $p < (n-r)$. then

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a static projective controller $\bar{u}_r = -K_{rs}\bar{y}_r$ will be computed. Then from (4.32b), $P_o = -K_{rs}^T$. The choice of performance criteria (4.34) is made in order to minimize z_r. However, the size of the gains used in the minimization must be limited since large gains can cause stability problems for projective controllers. Thus, for "small" R, z_r will be "small". For "large" R, K_{rs} will be "small". Thus, the goal is to choose R to make z_r small and to retain that property in the projective system.

Once P_o has been selected, the dynamic projective controller is determined. Also $M(s)$ determines the frequency properties of the closed-loop system through Theorem 4.1. Thus, the dynamic projective controller design is complete.

In summary, this chapter develops methods to analyze the effect of the choice of projective controller parameters on the frequency response of the system. In the case of the dynamic projective controller, a method is developed to assist in the choice of free parameter P_o. This completes the design of the projective controller.

5. THE CRUCIFORM MODEL

5.1. Description of the System.

The methods of the previous three chapters form a design procedure for disturbance rejection in which an arbitrary amount of disturbance rejection is achieved by a state feedback and implemented using projective controls. These methods will now be applied to achieve disturbance rejection in a cruciform structure. The cruciform is a 45 foot lattice-type, flexible beam which is attached at its base, but is free at its tip. It is light-weight (5 lbs.) and as a result, exhibits LSS characteristics. For a complete description, see Jones (1986). The system was modeled using a finite element analysis which was reduced to 20 modes. The resulting model has the form

$$\ddot{\eta} + 2\xi\Omega\dot{\eta} + \Omega^2\eta = B_L u(t) + D_L w(t) \tag{5.1}$$

where $\eta \in \mathbb{R}^{20}$ is the modal coordinate, $u \in \mathbb{R}^3$ is the control input, and $w \in \mathbb{R}^2$ is the disturbance input. The model parameters are located in Appendix A.

The control consists of torques applied at the base of the structure. The three controls apply torques about the three axes. The disturbance is generated by a hydraulically actuated table to which the base is mounted. The table is capable of translational motion in the x-y plane where the z-axis is taken to be the axis of the cruciform.

A number of measurements of the system are available for use in the control of the system. These measurements are obtained from 3-axis gyro and accelerometer sensors located at the tip and base of the structure. In terms of the modal coordinate,

$$y_r = C_r \dot{\eta}, \qquad y_r \in \mathbb{R}^6 \tag{5.2a}$$

$$y_a = C_a \ddot{\eta}, \qquad y_a \in \mathbb{R}^6. \tag{5.2b}$$

The controlled output is the output which will be regulated through the use of control. The position measurement at the location of the accelerometers has been chosen as the outputs to be regulated. In terms of the modal coordinate this is

$$z = C_a \eta, \qquad z \in \mathbb{R}^6. \tag{5.3}$$

A description of each of the system inputs and outputs is located in Tables A.1 and A.2 respectively located in Appendix A.

5.2. State Space Representation of the System.

The 2nd order matrix differential equation can be represented as a 40th order state space system of the form

$$\dot{x} = Ax + Bu + Dw \tag{5.4a}$$

$$y = Cx + Gu + Hw \tag{5.4b}$$

$$z = Ex \tag{5.4c}$$

where,

$$x \overset{\Delta}{=} \begin{bmatrix} \dot{\eta} \\ \eta \end{bmatrix}, \quad y \overset{\Delta}{=} \begin{bmatrix} y_r \\ y_a \end{bmatrix} \tag{5.5}$$

$$A = \begin{bmatrix} -2\xi\Omega & -\Omega^2 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B_L \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} D_L \\ 0 \end{bmatrix} \tag{5.6}$$

$$C = \begin{bmatrix} C_r & 0 \\ -2C_a\xi\Omega & -C_a\Omega^2 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ C_a B_L \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ C_a D_L \end{bmatrix} \tag{5.7}$$

$$E = \begin{bmatrix} 0 & C_a \end{bmatrix}. \tag{5.8}$$

By examining the eigenvalues of the cruciform system in Table 5.1. it can be seen that this structure exhibits LSS properties. The model is of high order and the low frequency modes of the system are lightly damped and closely packed.

It should be observed that due to the structure of the system, the control is only able to affect the position of the structure relative to the base. Also, the available measurements do not contain information about the absolute position of the structure, i.e., the rigid body modes are uncontrollable and unobservable. For this reason, we will not consider the rigid body modes in this analysis. Thus, they will be removed from subsequent models of the system.

The frequency response of the cruciform system (neglecting the rigid body modes) from each disturbance to the controlled outputs is given in Fig. 5.1. Since the response of both z_3 and z_6 to each of the disturbances is less than -100 dB, these outputs will not be used in the disturbance rejection analysis. Also, since the u_3 (z-axis) control acts on the torsional modes of the system, this input will be not used for disturbance rejection.

In addition, the design model will neglect the direct feedthrough terms of the control and disturbance in the measurement. These terms are observed to be small and thus are neglected.

Table 5.1. Cruciform System Modes.

| Mode Number	Frequency (rad/sec)	Damping Factor	Description
1	.00001	.000	rigid body y
2	.00025	.000	rigid body x
3	.00083	.000	rigid body torsion
4	.84942	.005	1st bending x
5	.87782	.020	1st bending y
6	2.2420	.005	1st torsion
7	7.0290	.005	cruciform bending y
8	7.2100	.005	cruciform bending x
9	7.3299	.005	cruciform bending x
10	7.6227	.005	cruciform bending y
11	7.7327	.005	cruciform bending y
12	7.9790	.020	cruciform bending x
13	8.0682	.050	cruciform bending y
14	8.5012	.016	2nd bending x
15	10.460	.019	2nd bending y
16	19.910	.020	3rd bending x
17	20.380	.005	2nd torsion
18	25.930	.020	3rd bending y
19	42.760	.011	4th bending x
20	46.100	.018	4th bending y
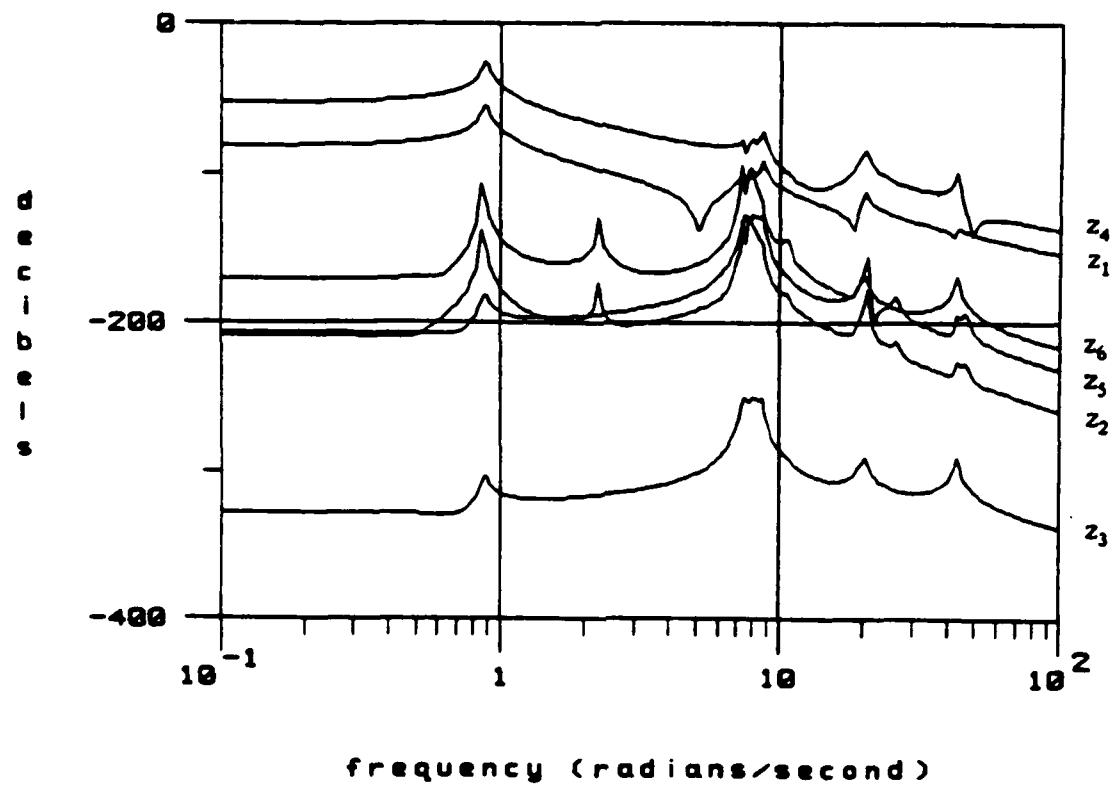
Figure 5.1a. Frequency Response of Cruciform System from w_1 to z.
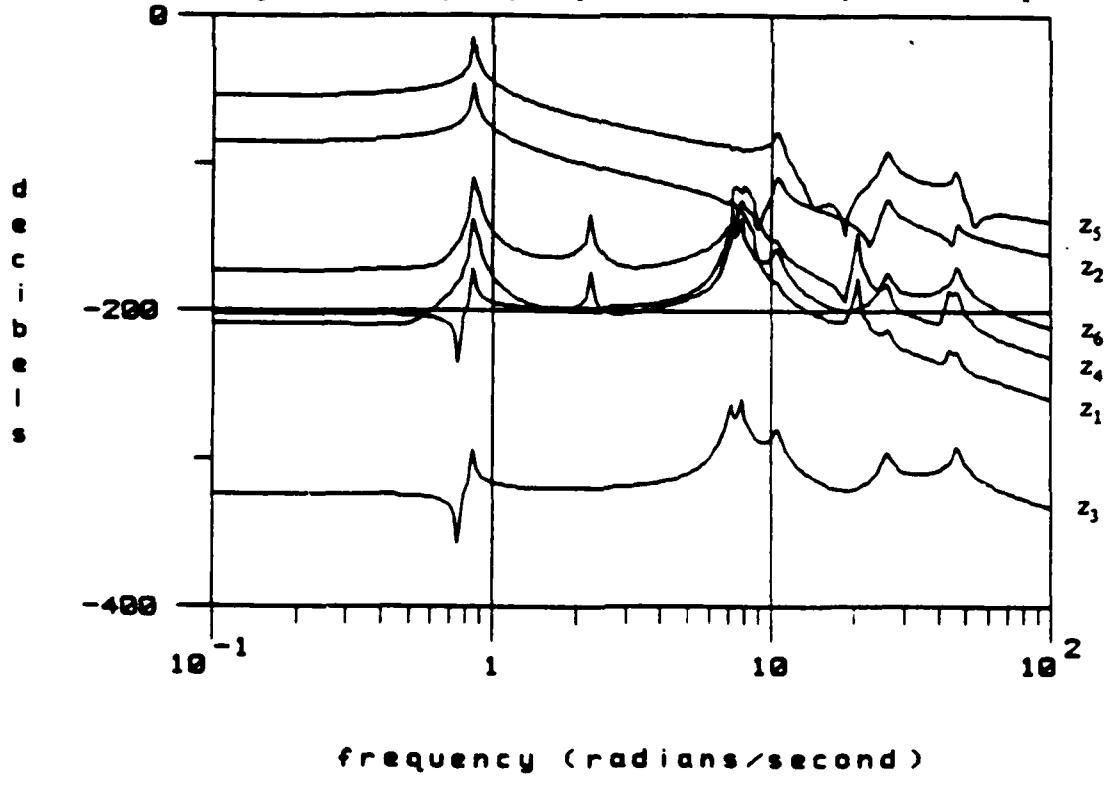
Figure 5.1b. Frequency Response of Cruciform System from w_2 to z.

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5.3. Generalized Hessenberg Representation.

The Generalized Hessenberg Representation (GHR) is a state-space realization of a linear system such that its input/output structure is made explicit (Tse, et al., 1978, Lindner and Perkins, 1982). It involves putting the system in a triangular form so that the interconnection of the system is made apparent. In this way, the strongly controllable (or observable) part of the system is separated from the weakly controllable (or observable) part of the system. More precisely, a system with state space representation (A,B,C) is transformed to a representation (F,G,H) where F, G and H have the following form.

$$
F = \begin{bmatrix}
F_{11} & F_{12} & 0 & 0 & \cdots & 0 & 0 \\
F_{21} & F_{22} & F_{23} & 0 & \cdots & 0 & 0 \\
F_{31} & F_{32} & F_{33} & F_{34} & \cdots & 0 & 0 \\
F_{41} & F_{42} & F_{43} & F_{44} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
F_{k-1,1} & F_{k-1,2} & F_{k-1,3} & F_{k-1,4} & \cdots & F_{k-1,k-1} & F_{k-1,k} \\
F_{k,1} & F_{k,2} & F_{k,3} & F_{k,4} & \cdots & F_{k,k-1} & F_{k,k}
\end{bmatrix}, \quad
G = \begin{bmatrix}
G_1 \\
G_2 \\
G_3 \\
G_4 \\
\vdots \\
G_{k-1} \\
G_k
\end{bmatrix}
\tag{5.9}
$$

$$
H = \begin{bmatrix} H_1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}
$$

where $F_{i,j} \in \mathbb{R}^{r_i \times r_i}$, $r_i > r_{i-1}$, and $r_1 = r$. Numerical issues associated with performing this transformation are found in Tharp (1983).

Let the system $(F_{i,i}, G_i, H_i)$ represent a subsystem of (F,G,H) called the i-th subsystem, then each $F_{i,j}$ block represents the connection from the j-th to the i-th subsystem. Due to the form of the GHR, if $F_{i-1,i} = 0$ then the 1st to (i-1)-th subsystems (hereafter called the upper subsystem) are simply cascaded into the i-th to k-th subsystems (hereafter called the lower subsystem). Since the output involves only states from the 1st subsystem, the lower subsystem is unobservable. In such a case, the modes of the upper subsystem are the observable modes of the system and the modes of the lower subsystem are the unobservable modes of the system.

If $F_{i-1,j}$ is not zero, but "small", then there exists a weak coupling between the upper and lower subsystems. In this case, the modes of the upper subsystem are the strongly observable modes of the system and the modes of the lower subsystem are the weakly observable modes of the system.

Since this division of modes by observability occurs for each $F_{i-1,j}$ block, the modes of each respective upper subsystem include progressively less observable modes. In this way, the modes of the system can be ordered by observability.

The system can also be transformed into a representation that displays the controllability of the system. In this case

$$
F = \begin{bmatrix}
F_{11} & F_{12} & F_{13} & \cdots & F_{1,k-1} & F_{1,k} \\
F_{21} & F_{22} & F_{23} & \cdots & F_{2,k-1} & F_{2,k} \\
0 & F_{32} & F_{33} & \cdots & F_{3,k-1} & F_{3,k} \\
0 & 0 & F_{43} & \cdots & F_{4,k-1} & F_{4,k} \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & \cdots & F_{k-1,k-1} & F_{k-1,k} \\
0 & 0 & 0 & \cdots & F_{k,k-1} & F_{k,k}
\end{bmatrix}, \quad
G = \begin{bmatrix}
G_1 \\
0 \\
0 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}
\qquad (5.10)
$$

$$
H = \begin{bmatrix} H_1 & H_2 & H_3 & \cdots & H_{k-1} & H_k \end{bmatrix}.
$$

where $F_{i,i} \in \mathbb{R}^{m_i \times m_i}$, $m_i > m_{i-1}$, and $m_1 = m$.

Here, $F_{i,i-1}$ provides the coupling between the upper and lower subsystems. If $F_{i,i-1}=0$, then the modes of the lower subsystem are uncontrollable. If $F_{i,i-1}$ is not zero, but "small", then the modes of the upper subsystem are the strongly controllable modes and the modes of the lower subsystem are the weakly controllable modes.

Using these properties, the GHR can be used as a method of determining reduced order, decentralized models for the system. The decentralization comes about as a result of the ability of the GHR to show weak interconnection between subsystems. Small $F_{i,i-1}$ blocks indicate a weak

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coupling of the upper and lower subsystems. Thus, the GHR has the ability to identify strongly coupled modes as part of a decentralized model. By applying the GHR to each input and output separately, an indication of the modes strongly coupled to each input and output is obtained. The decentralized model can be obtained by identifying groups of modes which are coupled to a group of inputs and outputs.

The GHR can also be used as a tool for model reduction. When performing model reduction, it becomes important to retain both the most controllable and most observable modes of the system. This is particularly true in LSS where the modes will easily go unstable.

In this way, model reduction and decentralization can be accomplished in one step. This is now done for the cruciform model.

5.4. Cruciform Decentralization Results.

In order to expose the input/output structure of the cruciform model, the GHR was applied to the system. Each input and output was used separately as the aggregation vector. The resulting transformed models were then analyzed for decoupling between subsystems. However, a difficulty arises in determining when two subsystems are weakly coupled. The norm of the $F_{i,i-1}$ can be used. However, it is not apparent that this is the best measure of subsystem coupling. Tharp (1987) has recently proposed that the coupling can be evaluated by the extent of the deviation of the modes resulting from assuming $F_{i,i-1}=0$. The smaller the deviation, the smaller the coupling. For details, see Tharp (1987).

An additional consideration is that the resulting decentralized models should include all modes important in determining disturbance rejection. Using Tharp's eigenvalue method, the cruciform system was analyzed. The results were very similar for each aggregation vector. In each case, two points of decoupling were noted. The first, denoted S1, possessed an upper subsystem of 2-3 modes. The other, denoted S2, possessed an upper subsystem of 9-10 modes. Table 5.2 gives the identifiable modes of the upper subsystem for both decoupling points.

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Table 5.2. Subsystem Modes of GHR Model.

Vector	S1	S2
u_1	20 18 15	20 19 18 17 16 15 4
u_2	19 16	20 19 18 17 16 14 12 9 5
w_1	19 16	20 19 18 17 16 14 12 9 5 2
w_2	20 18 15	20 19 18 17 16 15 13 11 4 1
y_1	20 18 15	20 19 18 17 16 15 13 11 7 4
y_2	19 16	20 19 18 17 16 14 12 11 9 5
y_3	17 6	
y_4	20 18 15	20 19 18 17 16 15 13 11 7 4
y_5	19 16	20 19 18 17 16 14 12 11 9 5
y_6	17 6	
y_7	19 16	20 19 18 17 16 14 12 9 5
y_8	20 18 15	20 19 18 17 16 15 13 11 7 4
y_9	20 19 18 16	20 19 18 17 16 15
y_{10}	19 16	20 19 18 17 16 14 12 11 9 5
y_{11}	20 18 15	20 19 18 17 16 15 4
y_{12}	20	20 19 18 16
z_1	19 16	20 19 18 17 16 14 12 5 2
z_2	20 18 15	20 19 18 17 16 15 4 1
z_3	19 16	20 19 18 17 16 14 12 9 5 2
z_4	20 18 15	20 19 18 17 16 15 4 1

Using these results, it is possible to develop decentralized models for the system. Since it is necessary to include the low frequency modes in the system model for disturbance rejection a model decentralization corresponding to S2 is considered. By grouping inputs and outputs by modes, a decentralized model is obtained. For the cruciform model, the resulting decentralized models are found in Table 5.3.

Table 5.3 GHR Decentralized Model Results.

Model	Inputs	Outputs	Modes
CRX	u_1,w_2	$z_2,z_5,y_1,y_4,y_8,y_{11}$	4 7 11 13 15 16 17 18 19 20
CRY	u_2,w_1	$z_1,z_4,y_2,y_5,y_7,y_{10}$	5 8 11 12 14 16 17 18 19 20

The frequency response of each of these models from disturbance to controlled output is given in Fig. 5.2. It can be seen that these models retain dominant features of the full model.

In summary, a model exhibiting LSS characteristics has been decomposed into two decentralized models using the GHR. These decentralized models form the design models for solving the

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frequency (radians/second)
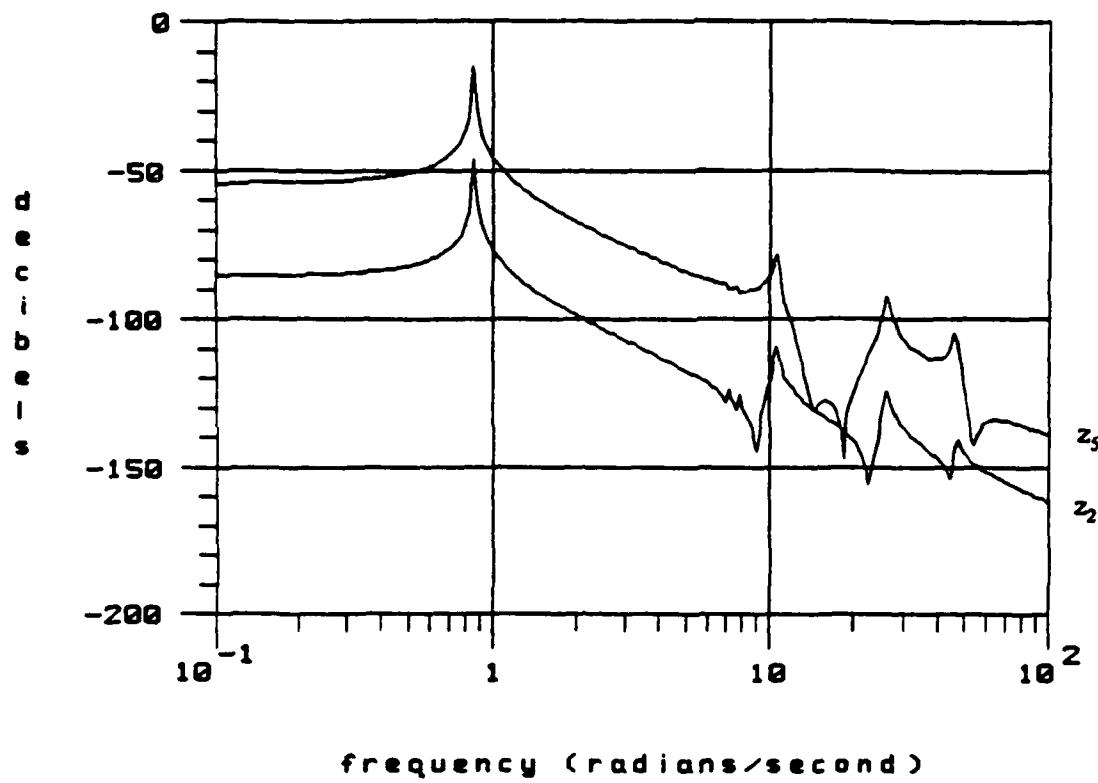
Figure 5.2a. Frequency Response of CRX system from w_2 to $z_2.z_4$.

frequency (radians/second)
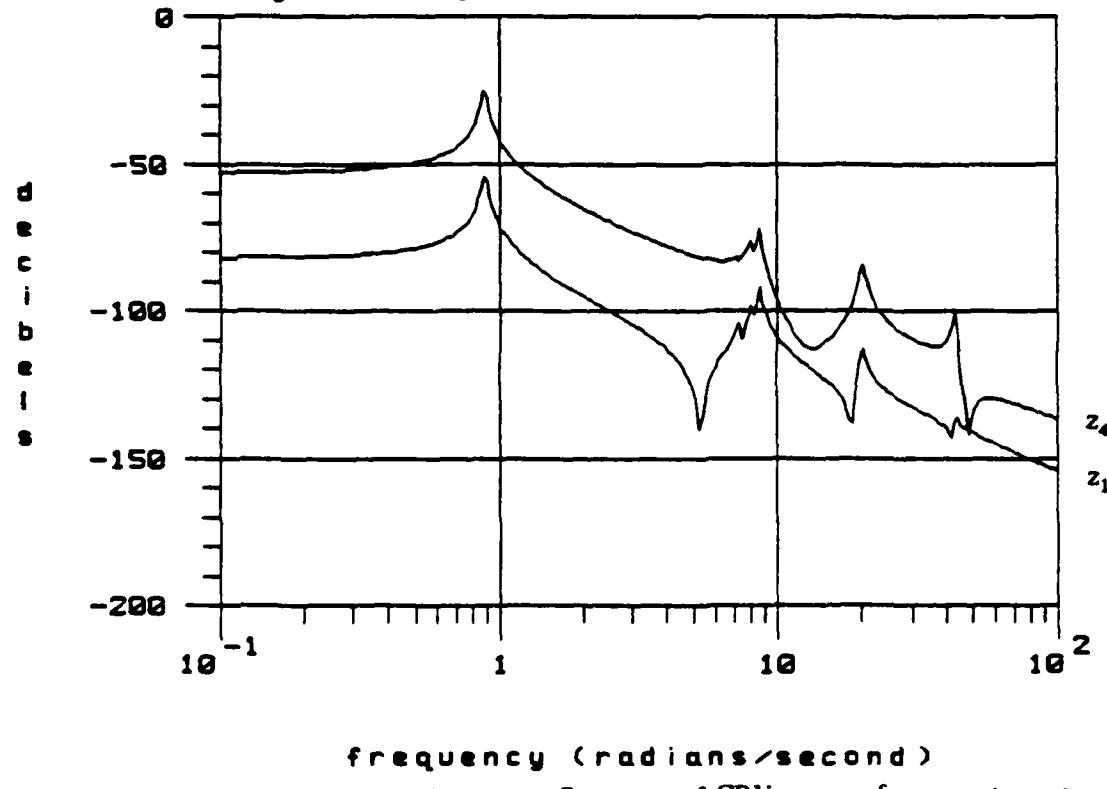
Figure 5.2b. Frequency Response of CRY system from w_1 to $z_1.z_3$.

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disturbance rejection problem for this system. Note that the amount of model reduction performed on the system is small. The model reduction which has been done on the system is mostly that which resulted from the decentralization. The reason for keeping a large design model is that it is desirable to keep any modes in the model that may be made unstable by the controller. The projective controls method allows one to keep a large order model without requiring a large order controller. Thus, a low order controller can be designed that accounts for all the modes present in the model and thus avoids spillover problems.

6. DISTURBANCE REJECTION FOR THE CRUCIFORM MODEL

6.1. Design Objectives.

In this chapter, an output feedback controller will be designed to improve the disturbance rejection properties of the cruciform model given in Chapter 5. Using the results of the previous chapter, the design will take a decentralized approach, i.e., a separate controller will be designed for each of the decentralized models and then combined to produce the overall controller. Fig. 6.1 shows the decentralized structure by which the group of outputs y1 are fed back to the controls u1 through K1 and likewise for y2 and u2. Each of the decentralized controllers will be designed in two steps. First, a state feedback solution will be computed for the decentralized model using the results of Chapter 2. Then, an output feedback solution will be calculated using the projective controls techniques of Chapter 3. The frequency properties of projective controllers established in Chapter 4 will then be used to choose the free parameters in the projective controllers. Both static and projective controls will be used in this example. Strictly proper projective controls will not be employed in this example, but could also be used to solve the problem.

Before proceeding with the design, the design criteria must be established. From Fig. 6.2, the nominal system has disturbance rejection of $\gamma = -15$ dB. This example will attempt to improve this to $\gamma = -40$ dB. In addition to the disturbance rejection goals, a design will be sought that is robust to parameter uncertainty and of low order.

Since the design of the controller is a decentralized one, the design goals of each system must be specified. From Fig. 6.3 it can be seen that the decentralized systems, CRX and CRY have a nominal disturbance rejection of $\gamma = -15$ dB and $\gamma = -25$ dB respectively. In order to obtain the system-wide goals and compensate for approximations made in the decentralization, disturbance

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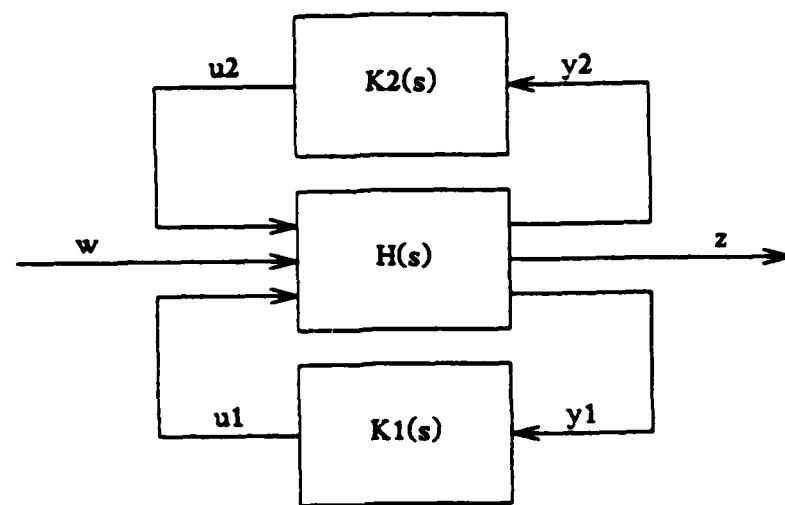
Figure 6.1. Decentralized Controller Structure.

frequency (radians/second)
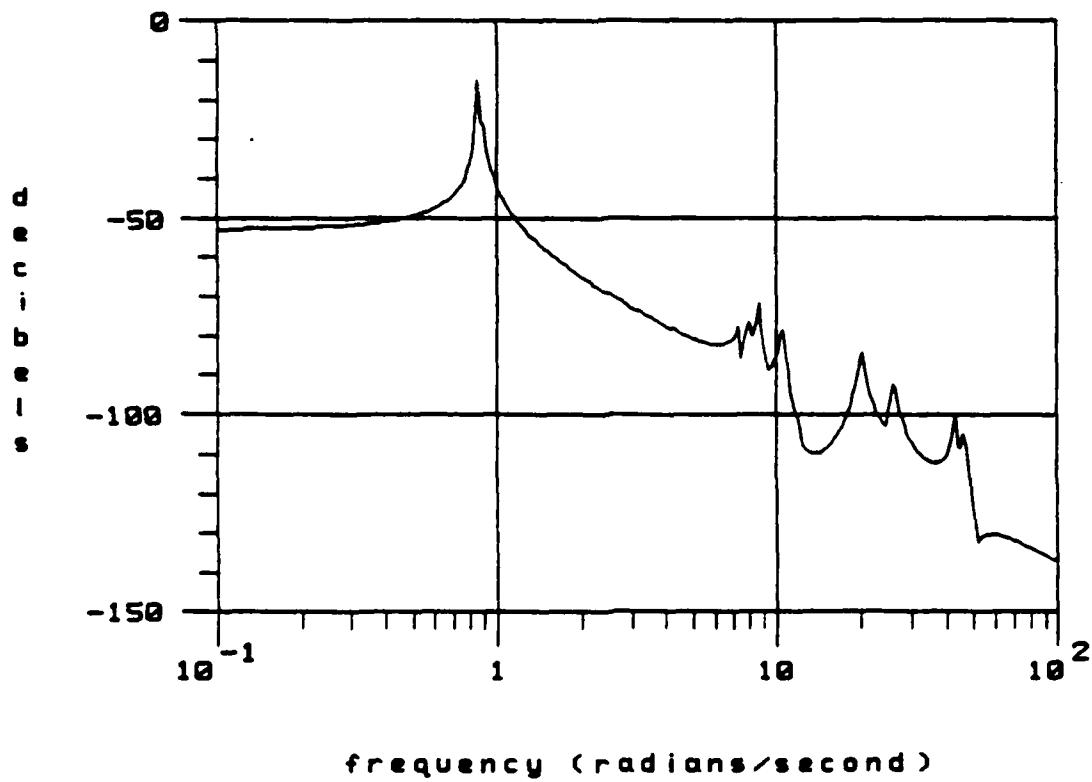
Figure 6.2. Maximum Singular Values of Cruciform System.

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frequency (radians/second)
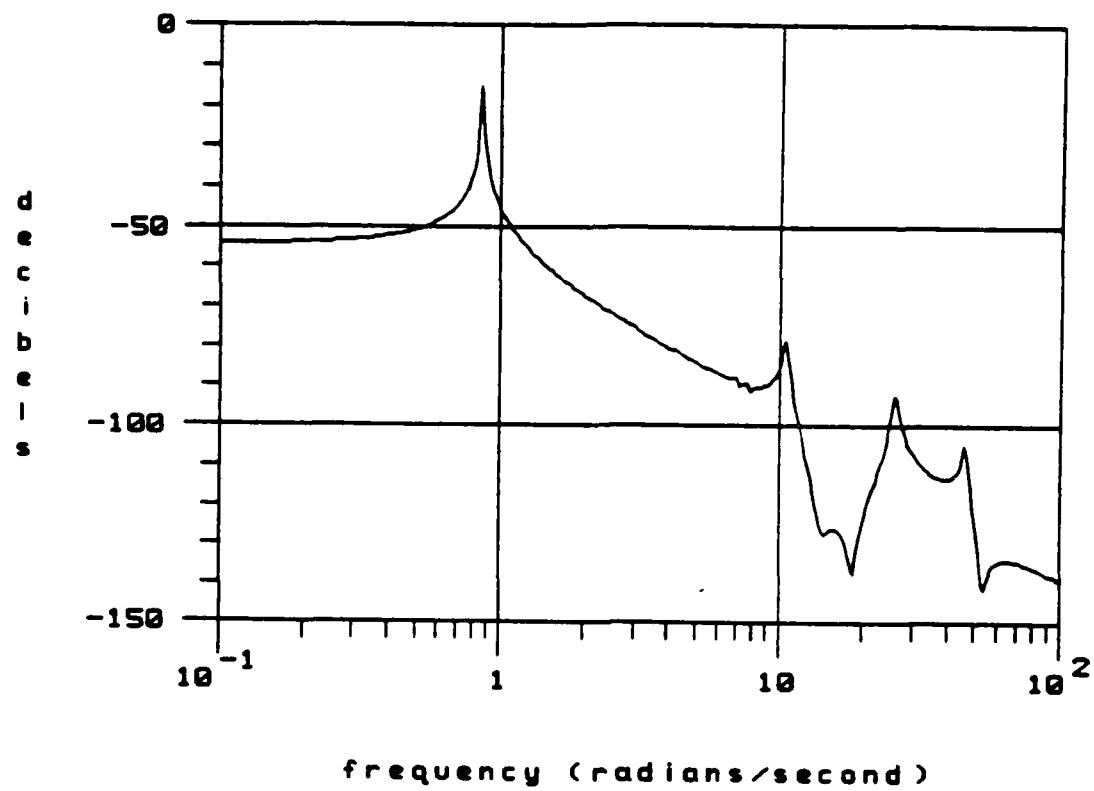
Figure 6.3a. Maximum Singular Values of CRX System.


frequency (radians/second)
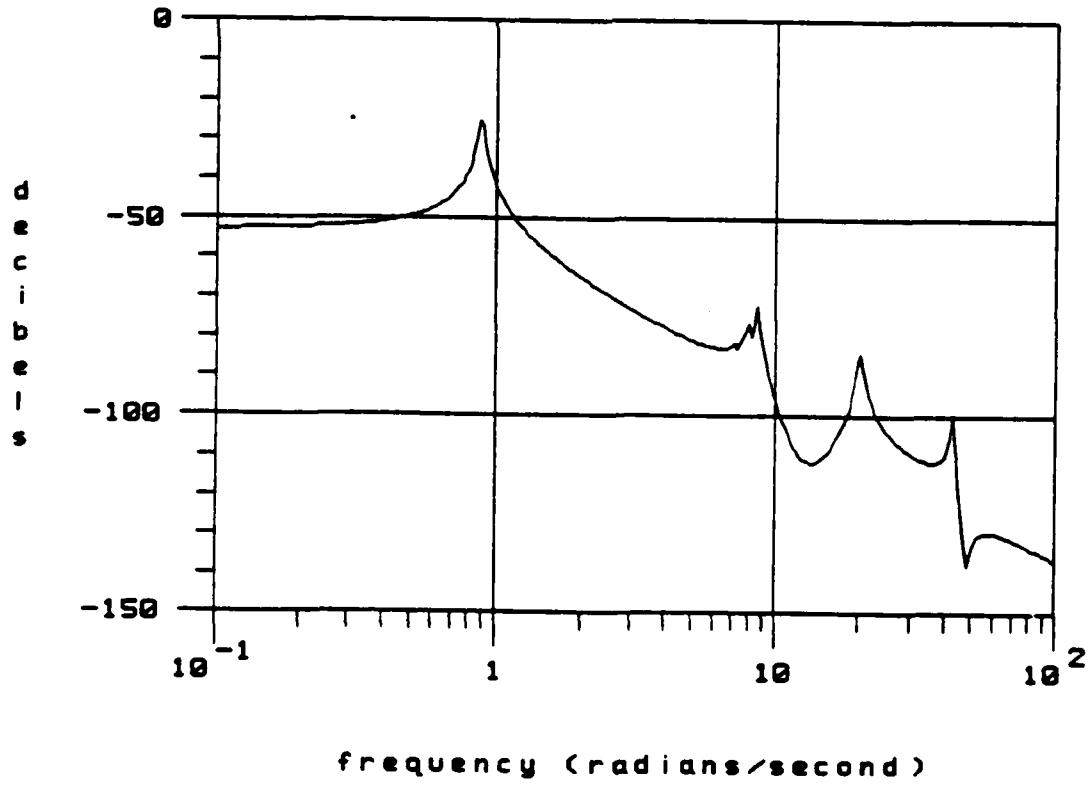
Figure 6.3b. Maximum Singular Values of CRY System.

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rejection of $\gamma = -45$ dB will be sought in each channel.

6.2. Full State Solution of the Disturbance Rejection Problem.

To solve the disturbance rejection problem for each model, Theorem 2.2 will be applied. This will yield a full-state solution to the problem. However, in order to apply the theorem, it is necessary that $R\{D\} \subseteq R\{B\}$. This is not true for either of the decentralized systems, but as will be seen, it is close enough to make an approximation. Consider another disturbance matrix, $D'=B\alpha$ where $\alpha = 0.34$ for the CRX system and $\alpha = 0.56$ for the CRY system. These values are chosen as the ratio of $\|E(sI-A)^{-1}D\|_\infty$ to $\|E(sI-A)^{-1}B\|_\infty$. Theorem 2.2 can now be applied to each system using D' and $\gamma = -45$ dB to yield a state feedback controller. The eigenvalues of the resulting closed loop system system, $\lambda(F)$ are given in Table 6.1.

In order to verify the disturbance properties of the actual system, this controller is applied to the system with the actual disturbance matrix. The disturbance rejection in this case is seen from Fig. 6.4 to be $\gamma = -45$ dB. Also, the system has gain margins of $0,\infty$ and phase margins of $\pm 85°$. Thus, this state feedback system solves the disturbance rejection problem and forms the reference system which will be approximated by a low order output feedback controller. Note that modes a_x and a_y are well damped by the applied control in order to achieve disturbance rejection.

Table 6.1a. Spectrum of CRX Reference System, $\lambda(F)$

| Mode	Real	Imaginary	Frequency	Damping
a_x	-1.2378e-01	±8.5697e-01	8.6586e-01	1.4296e-01
b_x	-3.5163e-02	±7.0290e+00	7.0290e+00	5.0025e-03
c_x	-3.8689e-02	±7.7327e+00	7.7328e+00	5.0033e-03
d_x	-4.0341e-01	±8.0581e+00	8.0682e+00	5.0000e-02
e_x	-2.1181e-01	±1.0459e+01	1.0461e+01	2.0248e-02
f_x	-3.9820e-01	±1.9906e+01	1.9910e+01	2.0000e-02
g_x	-1.0190e-01	±2.0380e+01	2.0380e+01	5.0000e-03
h_x	-5.2138e-01	±2.5925e+01	2.5930e+01	2.0107e-02
i_x	-4.7036e-01	±4.2757e+01	4.2760e+01	1.1000e-02
j_x	-8.3005e-01	±4.6092e+01	4.6100e+01	1.8006e-02
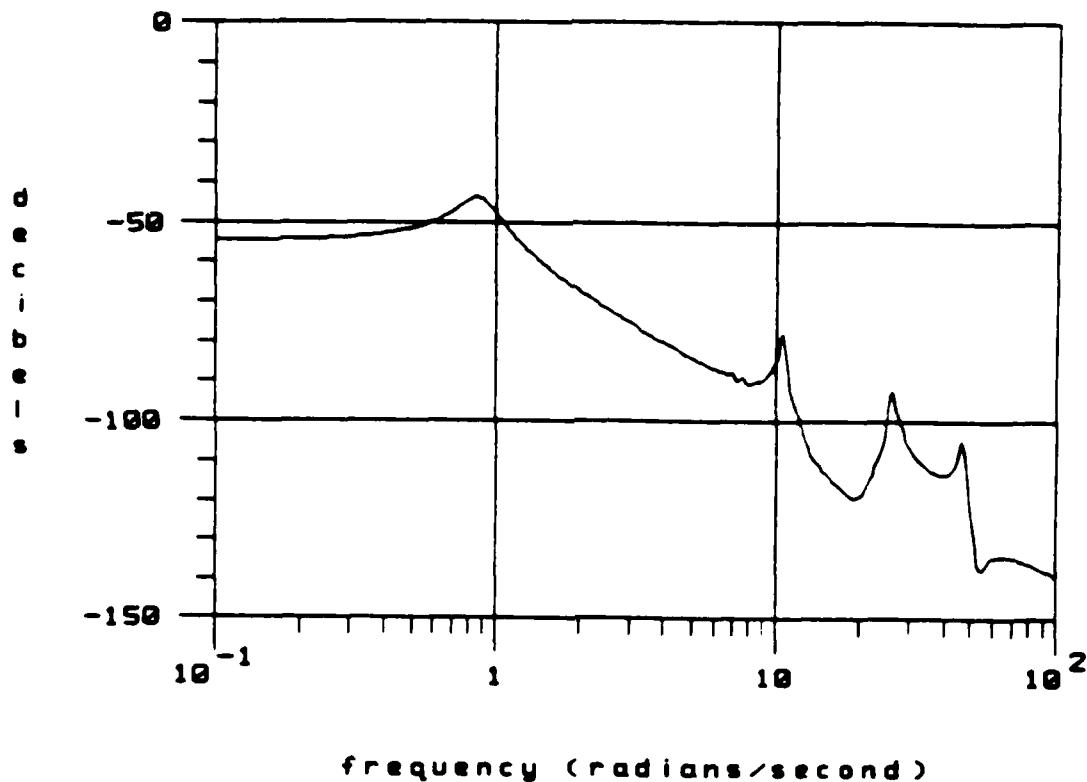
Figure 6.4a. Maximum Singular Values of CRX System with State Feedback.
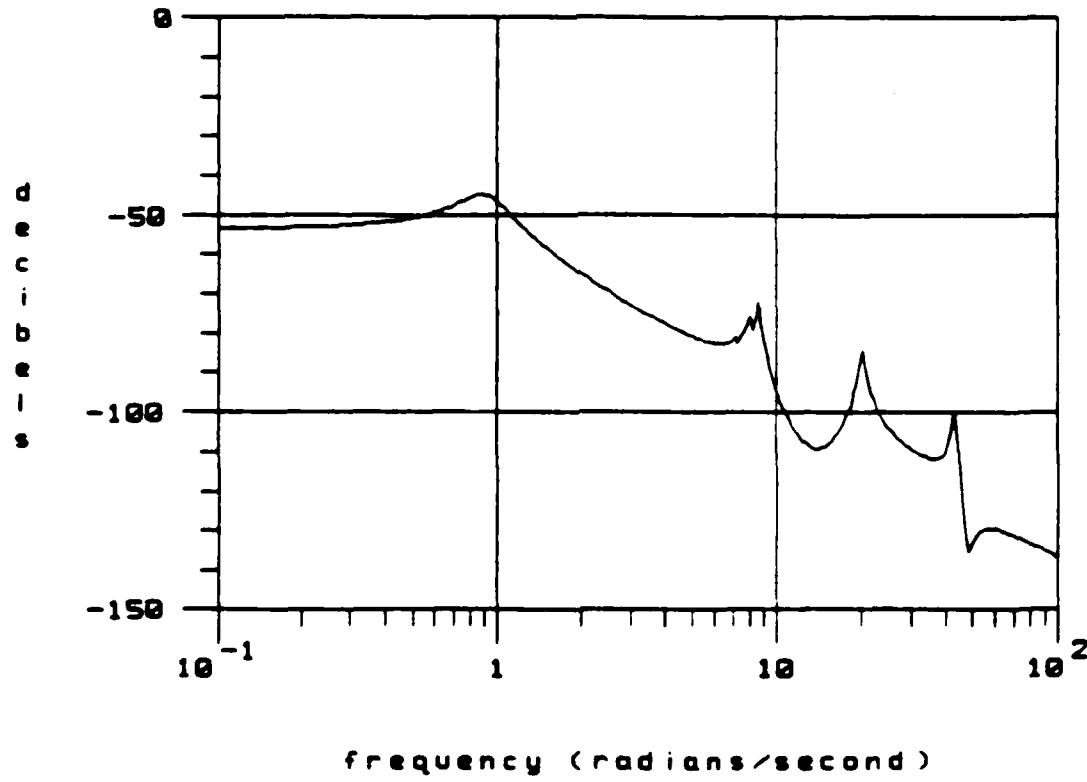
Figure 6.4b. Maximum Singular Values of CRY System with State Feedback.

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Table 6.1b. Spectrum of CRY Reference System, $\lambda(F)$

Mode	Real	Imaginary	Frequency	Damping
a_y	-1.7315e-01	±8.9171e-01	9.0836e-01	1.9062e-01
b_y	-3.6119e-02	±7.2100e+00	7.2101e+00	5.0095e-03
c_y	-3.8668e-02	±7.7326e+00	7.7327e+00	5.0006e-03
d_y	-1.6282e-01	±7.9796e+00	7.9812e+00	2.0401e-02
e_y	-1.4766e-01	±8.4982e+00	8.4994e+00	1.7373e-02
f_y	-4.0103e-01	±1.9906e+01	1.9910e+01	2.0142e-02
g_y	-1.0191e-01	±2.0380e+01	2.0380e+01	5.0003e-03
h_y	-5.1860e-01	±2.5925e+01	2.5930e+01	2.0000e-02
i_y	-4.7048e-01	±4.2757e+01	4.2760e+01	1.1003e-02
j_y	-8.2980e-01	±4.6092e+01	4.6100e+01	1.8000e-02

6.3. Static Projective Controls Solution.

First static projective controls will be applied to the disturbance rejection problem. Since the cruciform system has r=4 outputs. 2 modes can be retained in the projective system. As noted earlier. modes a_x and a_y are the dominant modes which affect disturbance rejection. Thus, these modes will be retained in the respective models. The choice of the other mode for retention is less clear. Thus, a projective controls solution will be calculated for each of the other modes. The results are found in Table 6.2 along with a calculation of the resulting systems' gain margin g_m and phase margin ϕ_m. For a particular choice of retained modes. these results may be satisfactory for some applications. However, improvement will be sought using dynamic projective controls.

Table 6.2a. CRX Static Projective Controls Results

λ_r	Disturbance Rejection	g_m	ϕ_m
$\{a_x, b_x\}$	$\gamma = -42$ dB	30 dB	90°
$\{a_x, c_x\}$	$\gamma = -42$ dB	30 dB	90°
$\{a_x, d_x\}$	h unstable	-	-
$\{a_x, e_x\}$	j unstable	-	-
$\{a_x, f_x\}$	h unstable	-	-
$\{a_x, g_x\}$	h unstable	-	-
$\{a_x, h_x\}$	$\gamma = -30$ dB	~0 dB	~0°
$\{a_x, i_x\}$	j unstable	-	-
$\{a_x, j_x\}$	$\gamma = -42$ dB	16 dB	80°

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Table 6.2b. CRY Static Projective Controls Results

λ_r	Disturbance Rejection	g_m	ϕ_m
$\{a_y, b_y\}$	i unstable	-	-
$\{a_y, c_y\}$	i unstable	-	-
$\{a_y, d_y\}$	d unstable	-	-
$\{a_y, e_y\}$	b. d. i unstable	-	-
$\{a_y, f_y\}$	$\gamma = -10$ dB	~0 dB	~0°
$\{a_y, g_y\}$	$\gamma = -30$ dB	~0 dB	~0°
$\{a_y, h_y\}$	f unstable	-	-
$\{a_y, i_y\}$	$\gamma = -42$ dB	5 dB	50°
$\{a_y, j_y\}$	$\gamma = -42$ dB	5 dB	50°

6.4. Dynamic Projective Controls Solution.

In order to improve upon the static design, a dynamic projective controller will now be considered. The order of the controller will be taken to be p=2. If this is found to be insufficient, the order of the controller may be increased.

In the design of the dynamic projective controller, there are two main design freedoms: the choice of the retained spectra λ_r and λ_p and the choice of the design parameter P_0. The choice of λ_r and λ_p is made to retain disturbance rejection properties and stability margins as much as possible. The modes a_x and a_y are retained in their respective systems as before in order to preserve the damping of these mode for disturbance rejection. The other two modes were chosen to improve and simplify the minimization of $M(s)$. Thus the following modes will be retained.

$$\text{CRX:} \qquad \lambda_r = \{e_x, h_x\}, \qquad \lambda_p = \{a_x\}$$

$$\text{CRY:} \qquad \lambda_r = \{d_y, f_y\}, \qquad \lambda_p = \{a_y\}$$

The nominal value of $M(s)$ (i.e. $P_0=0$) is given in Fig. 6.5. To choose P_0, the method of Chapter 4 is applied. R is taken to be of the form $R=\rho I$. The value of ρ is then varied to achieve stability and disturbance rejection properties. A value of $\rho=0.1$ and $\rho=0.05$ is chosen for the CRX and CRY models for this design problem. A static projective controller retained the lowest frequency modes of the system of (4.33). The resulting choice of P_0 is given in Appendix B, Table B.1 and yields $\|M(s)\|_{\infty} < -10$ dB as seen in Fig. 6.6.

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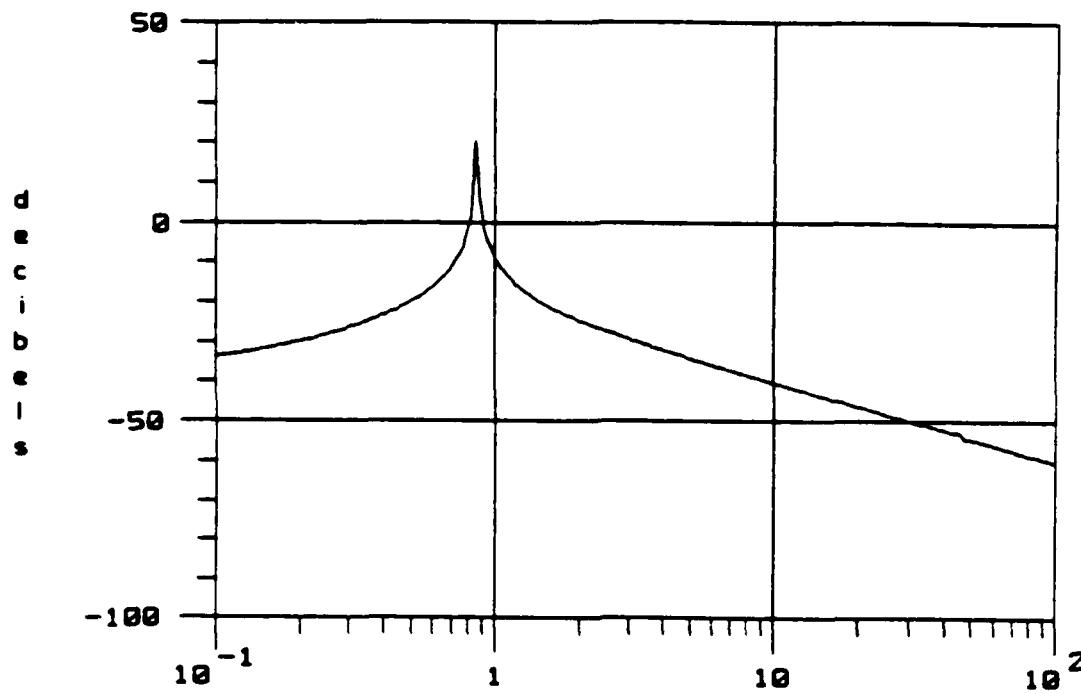
Figure 6.5a. $\|M(j\omega)\|$ for CRX System with $\lambda_r=\{e_x,h_x\}.\lambda_p=\{a_x\}.P_o=0$.
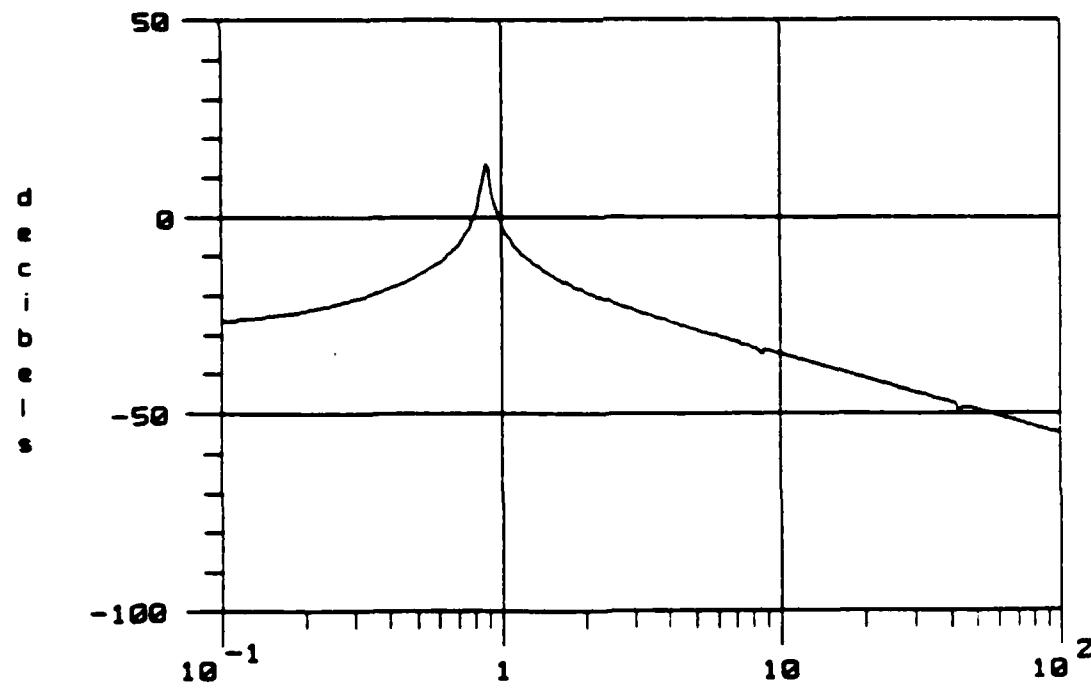
Figure 6.5b. $\|M(j\omega)\|$ for CRY System with $\lambda_r=\{d_y,f_y\}.\lambda_p=\{a_y\}.P_o=0$.

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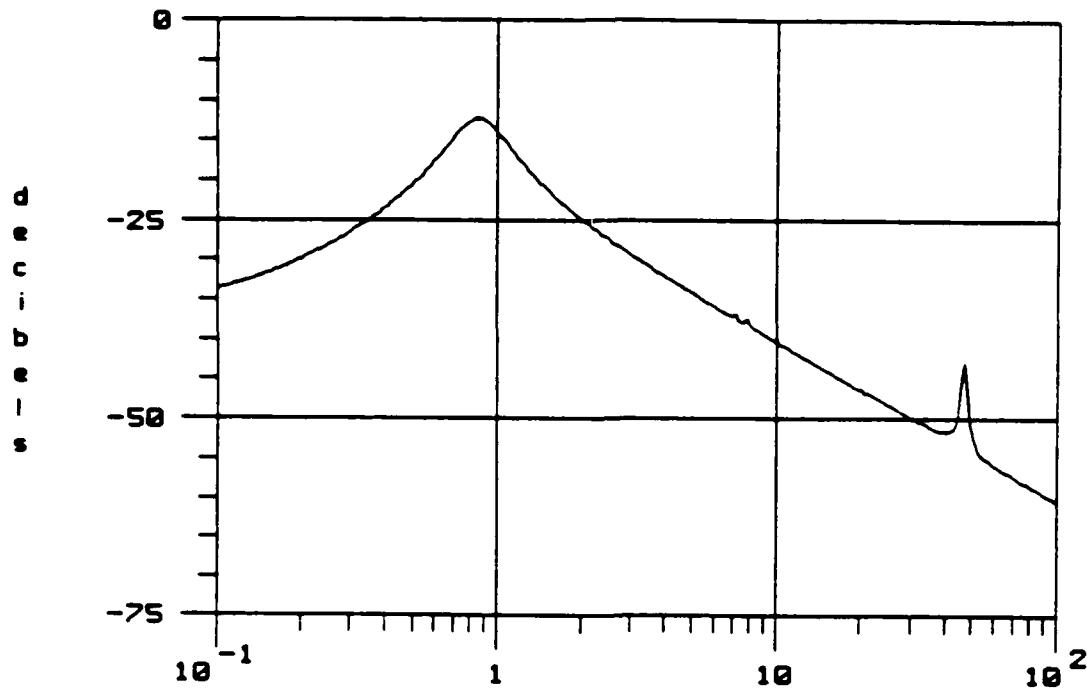
Figure 6.6a. $\|M(j\omega)\|$ for CRX System with P_o Optimized.
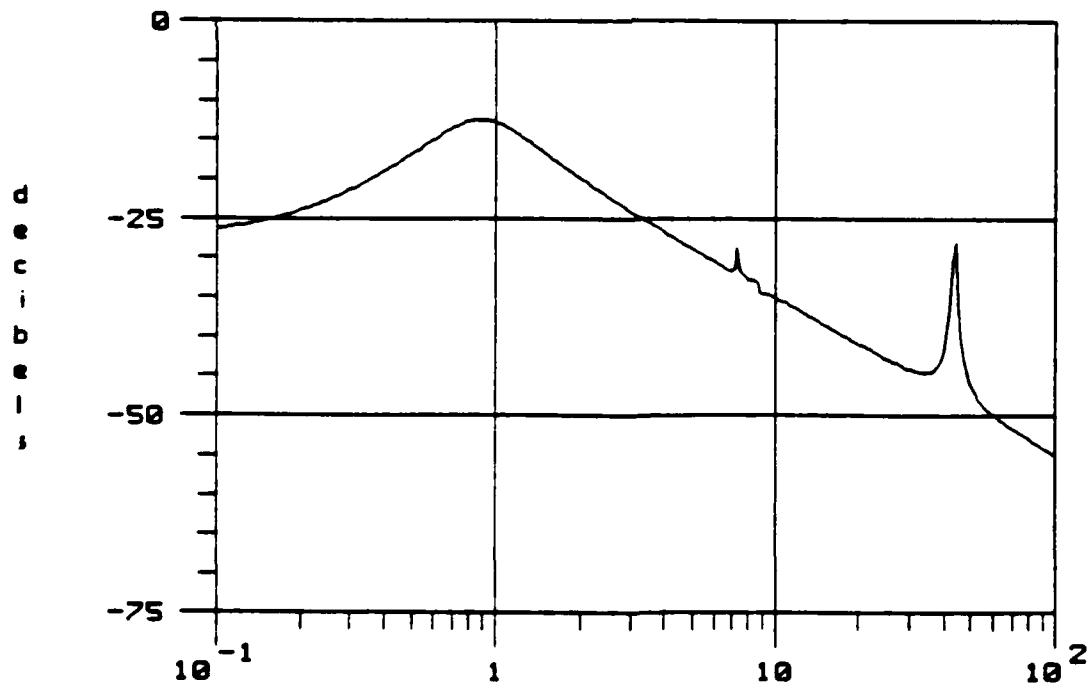
Figure 6.6b. $\|M(j\omega)\|$ for CRY System with P_o Optimized.

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The dynamic controllers associated with this choice of P_0 are given in Appendix B. Table B.2. The frequency responses of these controllers are given in Fig. 6.7. Both controllers can be seen to be essentially low-pass filters although this was not explicitly specified.

The closed-loop spectrum of the decentralized models for the above controllers is given in Table 6.3. Fig. 6.8 shows these controllers achieve disturbance rejection of $\gamma < -40$ dB.

Stability margins for this design are $g_m = 30$ dB and $\phi_m = 70°$ for each system. It may be observed, however, that a combined gain shift of 10 dB and phase shift of 30° will destabilize the system. This is not apparent from the stability margins and points to a need for better robustness

Table 6.3a. Spectrum of Dynamic Projective CRX System, $\lambda(A_{ce})$

Real	Imaginary	Frequency	Damping
-1.9565e-01	8.2765e-01	±8.5046e-01	2.3005e-01
-1.2378e-01	8.5697e-01	±8.6586e-01	1.4296e-01
-3.5019e-02	7.0255e+00	±7.0256e+00	4.9845e-03
-3.8596e-02	7.7297e+00	±7.7298e+00	4.9931e-03
-4.0340e-01	8.0572e+00	±8.0673e+00	5.0005e-02
-2.1181e-01	1.0459e+01	±1.0461e+01	2.0248e-02
-3.9821e-01	1.9906e+01	±1.9910e+01	2.0000e-02
-1.0190e-01	2.0380e+01	±2.0380e+01	5.0000e-03
-5.2138e-01	2.5925e+01	±2.5930e+01	2.0107e-02
-4.7036e-01	4.2757e+01	±4.2760e+01	1.1000e-02
-8.6824e-01	4.6660e+01	±4.6668e+01	1.8605e-02

Table 6.3b. Spectrum of Dynamic Projective CRY System, $\lambda(A_{ce})$

Real	Imaginary	Frequency	Damping
-3.7758e-01	7.9909e-01	±8.8381e-01	4.2722e-01
-1.7315e-01	8.9171e-01	±9.0836e-01	1.9062e-01
-3.5383e-02	7.1954e+00	±7.1955e+00	4.9174e-03
-3.8674e-02	7.7324e+00	±7.7325e+00	5.0015e-03
-1.6282e-01	7.9796e+00	±7.9812e+00	2.0401e-02
-1.5646e-01	8.6035e+00	±8.6049e+00	1.8183e-02
-4.0103e-01	1.9906e+01	±1.9910e+01	2.0142e-02
-1.0195e-01	2.0380e+01	±2.0380e+01	5.0023e-03
-5.1860e-01	2.5925e+01	±2.5930e+01	2.0000e-02
-4.9020e-01	4.3306e+01	±4.3309e+01	1.1319e-02
-8.2980e-01	4.6092e+01	±4.6100e+01	1.8000e-02
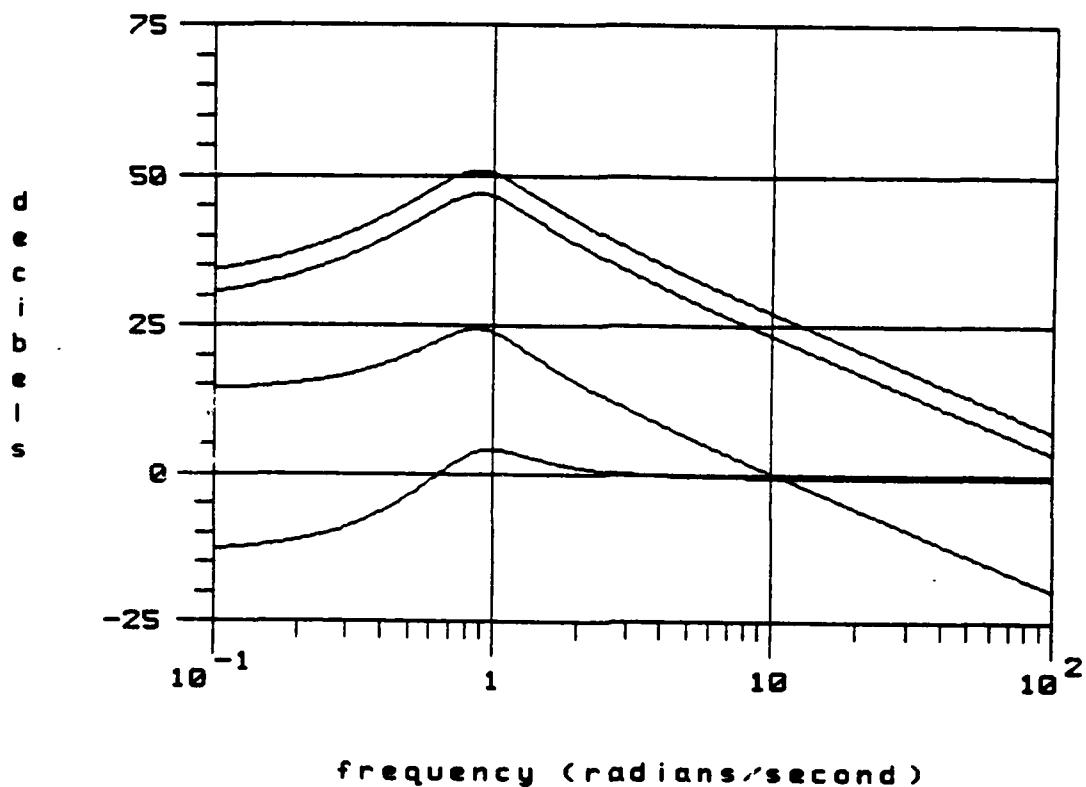
Figure 6.7a. Frequency Response of Dynamic Projective Controller for CRX System.
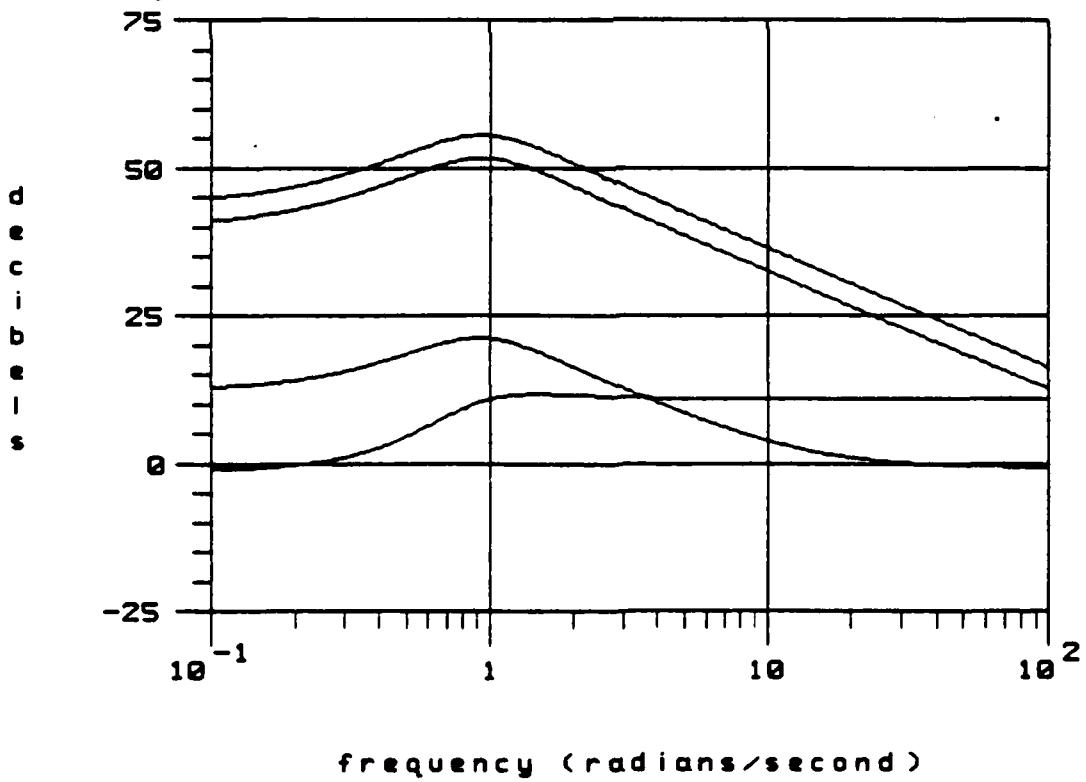
Figure 6.7b. Frequency Response of Dynamic Projective Controller for CRY System.

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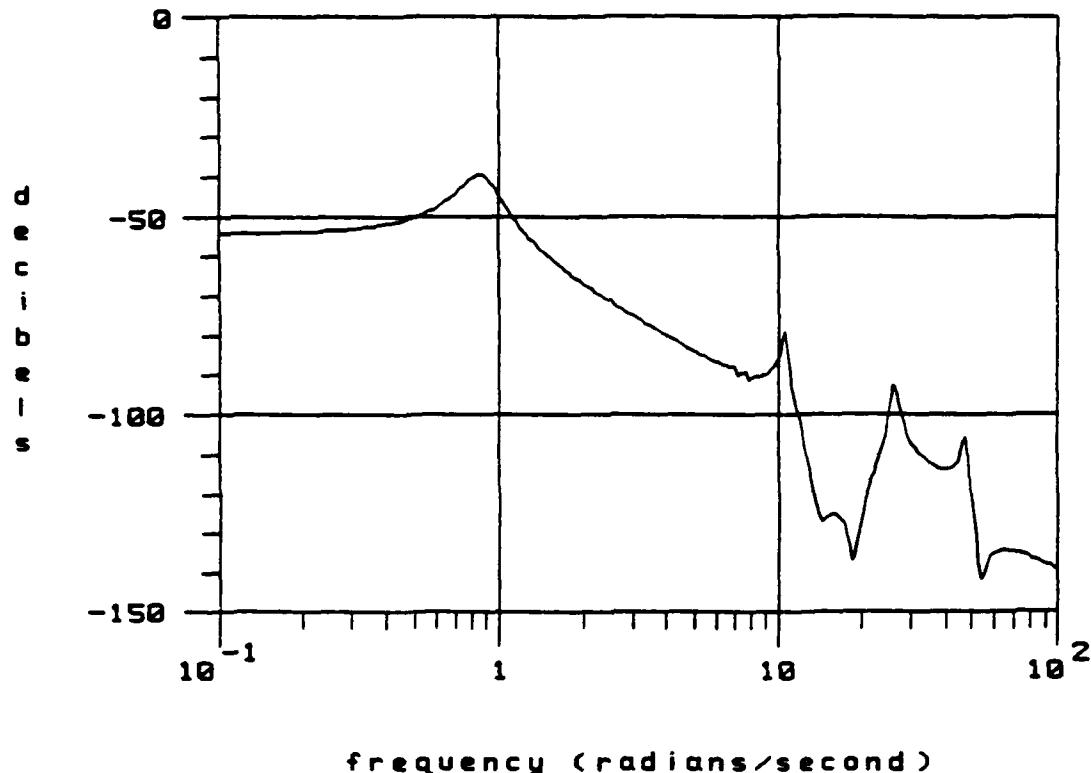
Figure 6.8a. Maximum Singular Values of CRX System with Dynamic Projective Controller.
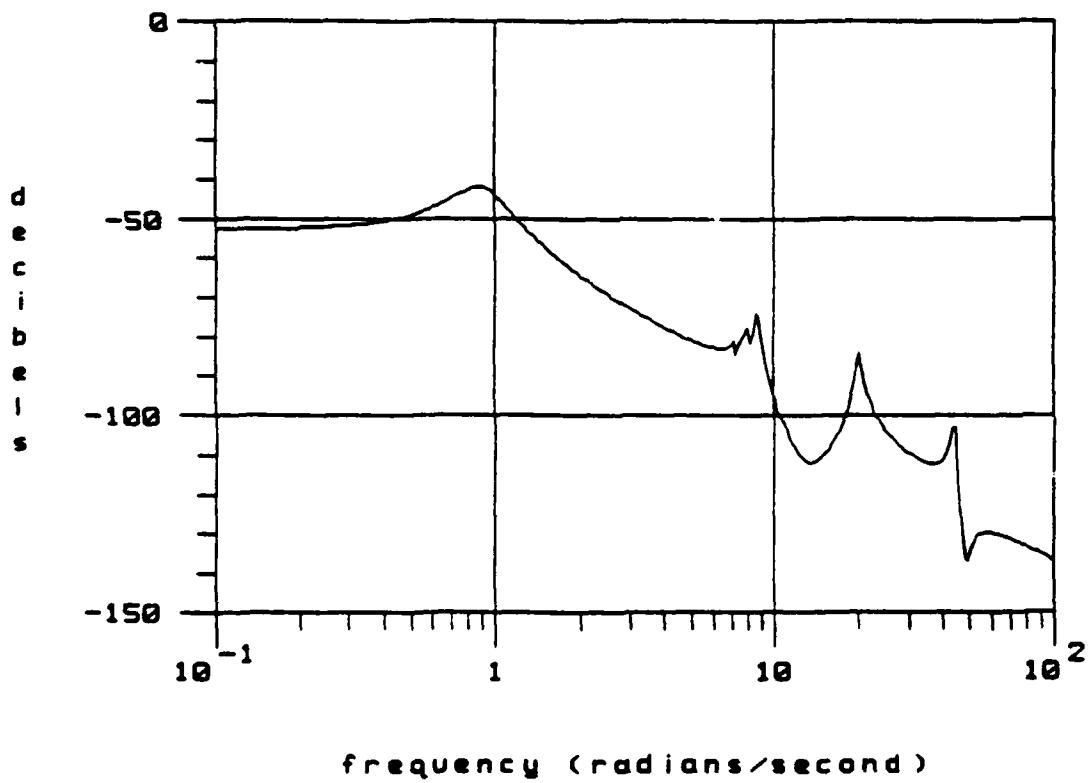
Figure 6.8b. Maximum Singular Values of CRY System with Dynamic Projective Controller.

measures.

Thus, each of the closed-loop decentralized systems possesses at least a modest amount of robustness. In addition, due to the low-pass nature of the dynamic controllers, less control energy is applied at high frequencies where uncertainty is the greatest, than in the static projective controller case.

6.5. Centralized Analysis of Decentralized Design.

The next step is to evaluate the design by applying the two decentralized controllers to the full system. The stability of the resulting closed loop system is shown in Table 6.4. The disturbance rejection of the full system is $\gamma = -40$ dB as seen in Fig. 6.9. Thus, the disturbance rejection goals of the design have been met. The stability margins of the system are $g_m = 70$ dB and $\phi_m = 80°$. If these stability margins are·not large enough, a higher order compensator or a different choice of retained spectrum may be considered.

Table 6.4. Spectrum of Closed Loop CRUC System.

Real	Imaginary	Frequency	Damping
-1.1239e-01	±8.4748e-01	8.5490e-01	1.?147e-01
-2.1821e-01	±8.4124e-01	8.6908e-01	2.5108e-01
-1.6552e-01	±8.8479e-01	9.0014e-01	1.8389e-01
-4.0343e-01	±8.0633e-01	9.0163e-01	4.4745e-01
-1.1211e-02	±2.2420e+00	2.2420e+00	5.0004e-03
-3.5007e-02	±7.0254e+00	7.0255e+00	4.9828e-03
-3.5147e-02	±7.1939e+00	7.1940e+00	4.8856e-03
-3.5896e-02	±7.3209e+00	7.3210e+00	4.9032e-03
-3.8220e-02	±7.6179e+00	7.6180e+00	5.0171e-03
-3.8602e-02	±7.7295e+00	7.7296e+00	4.9941e-03
-1.6281e-01	±7.9793e+00	7.9809e+00	2.0399e-02
-4.0385e-01	±8.0584e+00	8.0685e+00	5.0053e-02
-1.5713e-01	±8.6032e+00	8.6047e+00	1.8261e-02
-2.1189e-01	±1.0459e+01	1.0461e+01	2.0256e-02
-4.0106e-01	±1.9906e+01	1.9910e+01	2.0143e-02
-1.0195e-01	±2.0380e+01	2.0380e+01	5.0024e-03
-5.2140e-01	±2.5925e+01	2.5930e+01	2.0108e-02
-4.9087e-01	±4.3311e+01	4.3314e+01	1.1333e-02
-8.6879e-01	±4.6664e+01	4.6672e+01	1.8615e-02

frequency (radians/second)
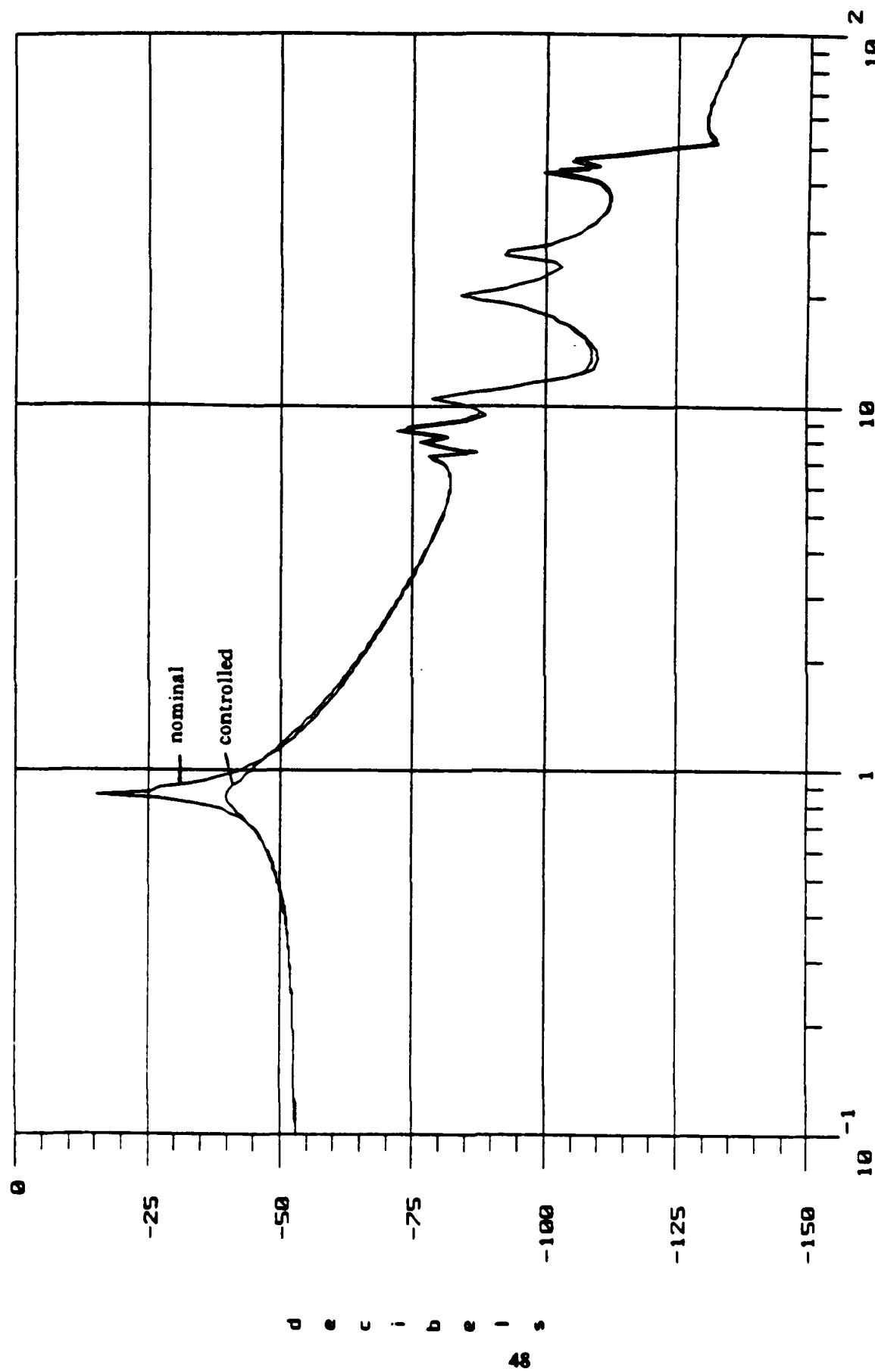
Figure 6.9. Maximum Singular Values of Cruciform System with Dynamic Projective Controller.

decibels

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In order to demonstrate the disturbance rejection achieved by this design, the time response of the system to a disturbance is computed. The disturbance considered is that of an impulse, i.e.,

$$w(t) = 100 \begin{vmatrix} \delta(t) \\ \delta(t) \end{vmatrix} \tag{6.1}$$

where $\delta(t)$ is the unit impulse function. The open loop response is given in Fig. 6.10. Note the low damping of the low frequency mode. For the system controlled by the design given above, the response is given in Fig. 6.11. In this case, the damping on the low frequency mode has increased dramatically. However, the high frequency dynamics are essentially unchanged. This is due to the fact that the design identified the low frequency mode as most affecting the response to the disturbance.

Thus, the design of this chapter reduces the response of the system to disturbances to a prescribed level. However, this level of disturbance rejection is achieved using a low order, robust controller as specified in the design goals.
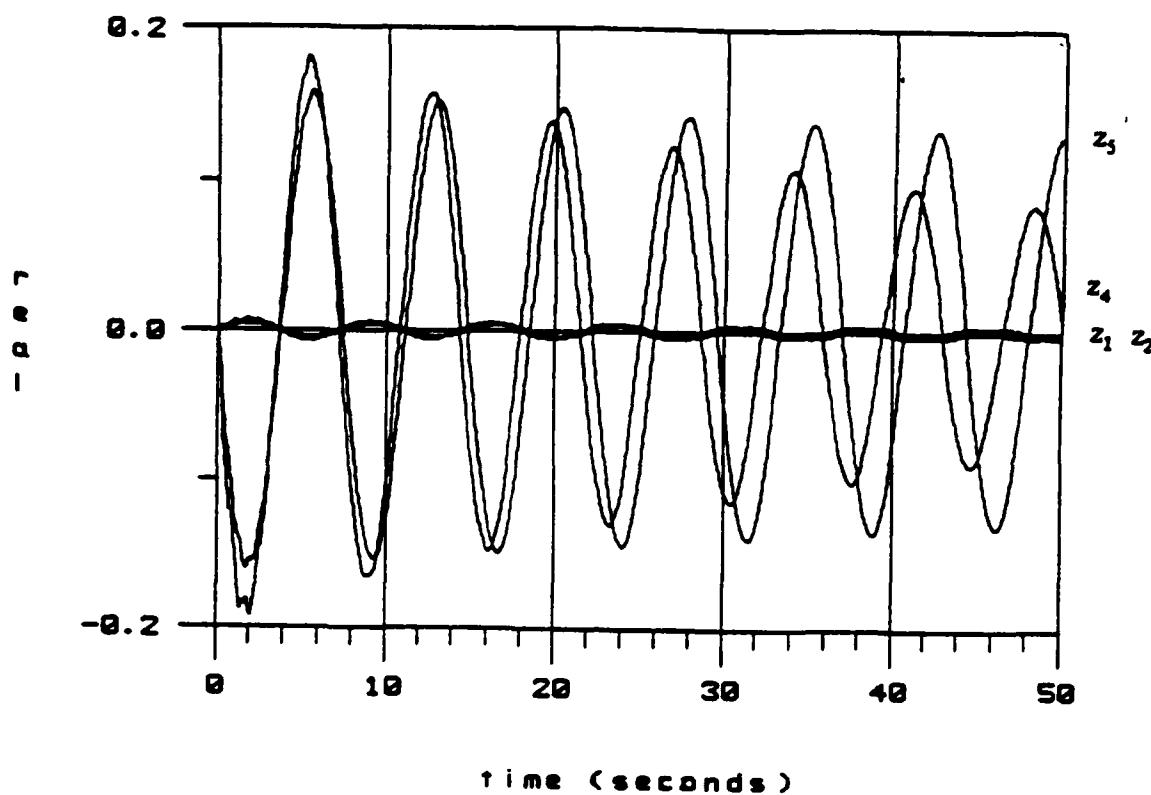
Figure 6.10. Time Response of Cruciform System.
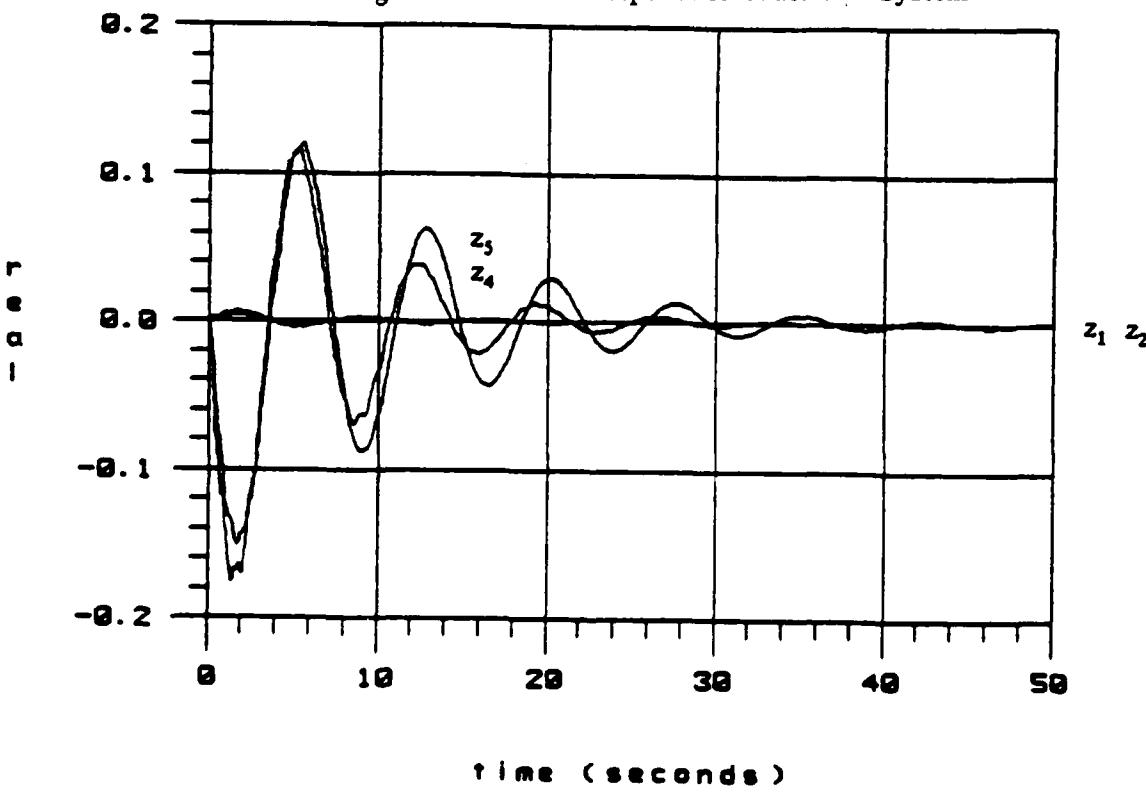
Figure 6.11. Time Response of Cruciform System with Dynamic Projective Controller.

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7. CONCLUSIONS

This thesis has presented a design methodology for improving the disturbance rejection of a system belonging to a class of systems called Large Space Structures. Properties of this class have been seen to include low modal damping and parameter uncertainty. Design problems associated with these properties have also been noted previously. The method of this thesis was constructed to deal with these difficulties in the context of the disturbance rejection problem.

Since the form of the disturbance is not precisely known, methods were used which accounted for this uncertainty. Through use of the H_∞-norm, given a knowledge of the "energy" (L_2-norm) of the disturbance, the "energy" of the system's response to the disturbance can be bounded. By applying control to the system, this bound can be reduced to a prescribed level using a state feedback controller.

The parameter uncertainty of the system prescribed a need for a robust controller. By using an LQ minimization approach, large stability margins are guaranteed. Thus, a state feedback controller is found which has desirable disturbance rejection and robustness properties.

Projective controls were then applied to the problem to design a low order compensator from the state feedback controller. In order to retain disturbance rejection properties, frequency domain properties of projective controllers were then developed. These properties were then used to select the free parameters of the projective controller.

This method was then applied to an example exhibiting LSS properties. The controller was determined in a decentralized manner using structural properties made apparent using the GHR. Using the method described above, a 25 dB improvement in the disturbance rejection of a 40th order LSS system was achieved using two decentralized second order controllers. In addition, a

degree of robustness was recovered from the state feedback controller.

Extensions to these concepts could be made for the case where frequency domain information is known about the disturbance. In such a case, frequency-weighting of the H_∞-norm and of the performance criteria of an LQ minimization may be applied. This would result in an extended state feedback controller. However, the projective controls techniques of this thesis could be applied without modification.

Thus, a design method for disturbance rejection of LSS systems has been shown. A design example has shown the application of this method for a system exhibiting LSS properties and produced a low order, robust controller solving the disturbance rejection problem.

APPENDIX A

Table A.1. Cruciform Model Parameters.

$$\Omega = \text{diag}
\begin{bmatrix}
2.513\text{e}{-}04 \\
1.3910\text{e}{-}05 \\
8.2850\text{e}{-}04 \\
8.4942\text{e}{-}01 \\
8.7782\text{e}{-}01 \\
2.2420\text{e}{-}00 \\
7.0290\text{e}{-}00 \\
7.2100\text{e}{-}00 \\
7.3300\text{e}{-}00 \\
7.6228\text{e}{-}00 \\
7.7327\text{e}{-}00 \\
7.9790\text{e}{-}00 \\
8.0682\text{e}{-}00 \\
8.5011\text{e}{-}00 \\
1.0460\text{e}{+}01 \\
1.9910\text{e}{+}01 \\
2.0380\text{e}{+}01 \\
2.5930\text{e}{+}01 \\
4.2760\text{e}{+}01 \\
4.6100\text{e}{+}01
\end{bmatrix}
\qquad
\xi = \text{diag}
\begin{bmatrix}
.000 \\
.000 \\
.000 \\
.005 \\
.020 \\
.005 \\
.005 \\
.005 \\
.005 \\
.005 \\
.005 \\
.020 \\
.005 \\
.016 \\
.019 \\
.020 \\
.005 \\
.020 \\
.011 \\
.018
\end{bmatrix}$$

$$B_L =
\begin{bmatrix}
7.144\text{e}{-}07 & 8.829\text{e}{-}13 & -3.946\text{e}{-}05 \\
-6.267\text{e}{-}14 & -2.170\text{e}{-}08 & -4.200\text{e}{-}05 \\
-1.651\text{e}{-}11 & 1.380\text{e}{-}11 & 2.224\text{e}{-}01 \\
-1.564\text{e}{-}02 & 2.570\text{e}{-}07 & -2.994\text{e}{-}04 \\
3.654\text{e}{-}07 & 1.385\text{e}{-}02 & -3.112\text{e}{-}04 \\
1.043\text{e}{-}04 & -1.007\text{e}{-}04 & -6.333\text{e}{-}02 \\
1.770\text{e}{-}02 & 4.871\text{e}{-}04 & 5.976\text{e}{-}06 \\
-2.549\text{e}{-}03 & -3.550\text{e}{-}02 & 3.897\text{e}{-}04 \\
-2.355\text{e}{-}03 & 3.588\text{e}{-}02 & 1.220\text{e}{-}04 \\
1.184\text{e}{-}02 & -2.322\text{e}{-}02 & 3.809\text{e}{-}04 \\
-1.925\text{e}{-}02 & -7.041\text{e}{-}03 & 2.500\text{e}{-}05 \\
-5.141\text{e}{-}03 & -7.797\text{e}{-}02 & -2.456\text{e}{-}04 \\
1.080\text{e}{-}02 & -2.006\text{e}{-}02 & 3.140\text{e}{-}04 \\
-5.175\text{e}{-}04 & 1.243\text{e}{-}01 & 3.521\text{e}{-}05 \\
-1.916\text{e}{-}01 & -2.810\text{e}{-}05 & 3.690\text{e}{-}05 \\
-7.178\text{e}{-}05 & 1.127\text{e}{-}01 & -1.329\text{e}{-}04 \\
-1.459\text{e}{-}03 & -5.674\text{e}{-}03 & -2.518\text{e}{-}03 \\
2.716\text{e}{-}01 & -2.096\text{e}{-}06 & -9.656\text{e}{-}06 \\
-9.120\text{e}{-}06 & -4.698\text{e}{-}02 & 7.325\text{e}{-}06 \\
-1.770\text{e}{-}01 & 2.299\text{e}{-}06 & -9.101\text{e}{-}06
\end{bmatrix}
\qquad
D_L =
\begin{bmatrix}
1.417\text{e}{-}09 & 3.503\text{e}{-}02 \\
3.807\text{e}{-}02 & -6.824\text{e}{-}11 \\
-3.549\text{e}{-}07 & -1.698\text{e}{-}08 \\
-1.522\text{e}{-}07 & -6.110\text{e}{-}03 \\
-7.738\text{e}{-}03 & 1.342\text{e}{-}07 \\
1.393\text{e}{-}05 & 7.851\text{e}{-}06 \\
-3.123\text{e}{-}05 & 4.292\text{e}{-}04 \\
2.258\text{e}{-}03 & -5.985\text{e}{-}05 \\
-2.273\text{e}{-}03 & -5.487\text{e}{-}05 \\
1.449\text{e}{-}03 & 2.707\text{e}{-}04 \\
4.377\text{e}{-}04 & -4.396\text{e}{-}04 \\
4.806\text{e}{-}03 & -1.141\text{e}{-}04 \\
1.234\text{e}{-}03 & 2.386\text{e}{-}04 \\
-7.523\text{e}{-}03 & -1.114\text{e}{-}05 \\
1.616\text{e}{-}06 & -3.771\text{e}{-}03 \\
-5.869\text{e}{-}03 & -1.181\text{e}{-}06 \\
2.946\text{e}{-}04 & -2.389\text{e}{-}05 \\
1.052\text{e}{-}07 & 4.242\text{e}{-}03 \\
2.175\text{e}{-}03 & -1.293\text{e}{-}07 \\
-1.050\text{e}{-}07 & -2.473\text{e}{-}03
\end{bmatrix}$$

$$
C_r^T = \begin{vmatrix}
7.144e{-}07 & 0.000e{+}00 & -3.946e{-}05 & 9.562e{-}07 & 0.000e{+}00 & -3.949e{-}05 \\
0.000e{+}00 & -2.170e{-}08 & -4.200e{-}05 & 0.000e{+}00 & -5.435e{-}08 & -4.200e{-}05 \\
0.000e{+}00 & 0.000e{+}00 & 2.224e{-}01 & 0.000e{+}00 & 0.000e{+}00 & 2.224e{-}01 \\
-1.564e{-}02 & 2.570e{-}07 & -2.994e{-}04 & -1.710e{-}02 & 3.323e{-}07 & 2.358e{-}04 \\
3.654e{-}07 & 1.385e{-}02 & -3.112e{-}04 & 3.967e{-}07 & 1.778e{-}02 & 2.829e{-}04 \\
1.043e{-}04 & -1.007e{-}04 & -6.333e{-}02 & 5.561e{-}05 & -6.235e{-}05 & 7.231e{-}01 \\
1.770e{-}02 & 4.871e{-}04 & 5.976e{-}06 & -2.657e{-}02 & -4.646e{-}04 & -7.021e{-}04 \\
-2.549e{-}03 & -3.550e{-}02 & 3.897e{-}04 & 2.387e{-}03 & 3.479e{-}02 & -4.811e{-}02 \\
-2.355e{-}03 & 3.588e{-}02 & 1.220e{-}04 & 2.399e{-}03 & -3.917e{-}02 & -1.555e{-}02 \\
1.184e{-}02 & -2.322e{-}02 & 3.809e{-}04 & -1.353e{-}02 & 1.884e{-}02 & -5.238e{-}02 \\
-1.925e{-}02 & -7.041e{-}03 & 2.500e{-}05 & 2.752e{-}02 & 5.873e{-}03 & -3.534e{-}03 \\
-5.141e{-}03 & -7.797e{-}02 & -2.456e{-}04 & 4.962e{-}03 & 6.735e{-}02 & 3.689e{-}02 \\
1.080e{-}02 & -2.006e{-}02 & 3.140e{-}04 & -1.078e{-}02 & 1.835e{-}02 & -4.819e{-}02 \\
-5.175e{-}04 & 1.243e{-}01 & 3.521e{-}05 & 5.377e{-}04 & -8.600e{-}02 & -5.972e{-}03 \\
-1.916e{-}01 & -2.801e{-}05 & 3.690e{-}05 & 1.823e{-}01 & 7.747e{-}06 & -9.256e{-}03 \\
-7.178e{-}05 & 1.127e{-}01 & -1.329e{-}04 & -3.891e{-}05 & 2.829e{-}01 & 9.886e{-}02 \\
-1.459e{-}03 & -5.674e{-}03 & -2.518e{-}03 & -8.886e{-}04 & -1.464e{-}02 & 1.937e{+}00 \\
2.716e{-}01 & -2.096e{-}06 & -9.656e{-}06 & 2.962e{-}01 & -5.066e{-}06 & 9.887e{-}03 \\
-9.120e{-}06 & -4.698e{-}02 & 7.325e{-}06 & 1.022e{-}05 & 3.637e{-}01 & -6.133e{-}03 \\
-1.770e{-}01 & 2.299e{-}06 & -9.101e{-}06 & 3.388e{-}01 & -2.373e{-}05 & 5.256e{-}03
\end{vmatrix}
$$

$$
C_a^T = \begin{vmatrix}
1.417e{-}09 & 3.503e{-}02 & 3.468e{-}23 & 1.432e{-}09 & 3.502e{-}02 & 4.663e{-}17 \\
3.807e{-}02 & 0.000e{+}00 & -1.463e{-}24 & 3.807e{-}02 & 0.000e{+}00 & -2.187e{-}18 \\
-3.549e{-}07 & -1.698e{-}08 & -1.329e{-}26 & -3.546e{-}07 & -1.673e{-}08 & 1.440e{-}21 \\
-1.522e{-}07 & -6.110e{-}03 & 2.384e{-}15 & 4.119e{-}06 & 2.222e{-}01 & 3.055e{-}09 \\
-7.738e{-}03 & 1.342e{-}07 & -2.582e{-}15 & 2.218e{-}01 & -5.184e{-}06 & -3.310e{-}09 \\
1.393e{-}05 & 7.851e{-}06 & 4.611e{-}18 & -1.267e{-}03 & -1.195e{-}03 & 5.910e{-}12 \\
-3.123e{-}05 & 4.292e{-}04 & 1.387e{-}10 & 7.993e{-}05 & 1.406e{-}02 & 1.778e{-}04 \\
2.258e{-}03 & -5.985e{-}05 & -3.414e{-}11 & 7.045e{-}03 & 4.497e{-}03 & -4.376e{-}05 \\
-2.273e{-}03 & -5.487e{-}05 & 1.290e{-}10 & -3.180e{-}02 & 3.039e{-}03 & 1.653e{-}04 \\
1.449e{-}03 & 2.707e{-}04 & 9.995e{-}12 & -1.190e{-}03 & -6.003e{-}03 & 1.274e{-}05 \\
4.377e{-}04 & -4.396e{-}04 & 2.020e{-}10 & 1.456e{-}03 & -1.779e{-}02 & 2.588e{-}04 \\
4.806e{-}03 & -1.141e{-}04 & 1.740e{-}10 & 5.317e{-}02 & 5.608e{-}03 & 2.230e{-}04 \\
1.234e{-}03 & 2.386e{-}04 & 8.846e{-}11 & 2.096e{-}02 & -9.427e{-}03 & 1.134e{-}04 \\
-7.523e{-}03 & -1.114e{-}05 & 9.249e{-}11 & -7.424e{-}02 & 1.910e{-}04 & 1.185e{-}04 \\
1.616e{-}06 & -3.771e{-}03 & -1.110e{-}11 & 4.036e{-}05 & -1.311e{-}01 & -1.423e{-}05 \\
-5.869e{-}03 & -1.181e{-}06 & -7.828e{-}12 & 1.680e{-}01 & -2.111e{-}05 & -1.002e{-}05 \\
2.946e{-}04 & -2.389e{-}05 & 4.332e{-}13 & -9.449e{-}03 & -3.073e{-}04 & 5.543e{-}07 \\
1.052e{-}07 & 4.242e{-}03 & -1.050e{-}11 & -6.058e{-}06 & -1.686e{-}01 & -1.343e{-}05 \\
2.175e{-}03 & -1.293e{-}07 & -5.344e{-}11 & 1.906e{-}01 & -3.141e{-}06 & -6.801e{-}05 \\
-1.050e{-}07 & -2.473e{-}03 & -8.271e{-}11 & -1.476e{-}05 & -1.763e{-}01 & -1.051e{-}04
\end{vmatrix}
$$

Table A.2. Cruciform System Inputs.

Input	Axis	Description
u_1	x	control
u_2	y	control
u_3	z	control
w_1	x	disturbance
w_2	y	disturbance

Table A.3. Cruciform System Outputs.

Output	Axis	Description
z_1	x	base displacement
z_2	y	base displacement
z_3	z	base displacement
z_4	x	tip displacement
z_5	y	tip displacement
z_6	z	tip displacement
y_1	x	base gyro
y_2	y	base gyro
y_3	z	base gyro
y_4	x	tip gyro
y_5	y	tip gyro
y_6	z	tip gyro
y_7	x	base acceleration
y_8	y	base acceleration
y_9	z	base acceleration
y_{10}	x	tip acceleration
y_{11}	y	tip acceleration
y_{12}	z	tip acceleration

APPENDIX B

Table B.1a. Free Parameter P_o for CRX Model.

$$P_o = \begin{vmatrix} 1.0376e{-}03 & 1.2024e{-}03 & -1.7464e{-}04 & 7.1238e{-}03 \\ 1.0895e{-}03 & 5.1881e{-}04 & 6.1253e{-}04 & -2.4600e{-}02 \end{vmatrix}$$

Table B.1b. Free Parameter P_o for CRY Model.

$$P_o = \begin{vmatrix} 1.9482e{-}03 & 4.8337e{-}03 & -6.8981e{-}05 & 4.7694e{-}04 \\ 3.8132e{-}03 & 1.1677e{-}03 & -2.9062e{-}03 & 7.3579e{-}02 \end{vmatrix}$$

Table B.2a. Parameters of Dynamic Projective Controller for CRX Model.

$$\begin{vmatrix} H & D \\ N_d & K_d \end{vmatrix} =$$

-3.2864e-01	8.1786e-01	3.3326e-01	5.2116e-01	1.3658e-01	2.6865e-02
-8.1786e-01	-3.2864e-01	-5.8299e+00	-9.0933e+00	-3.7198e-01	1.5239e-02
6.8014e+00	-2.4218e+01	4.4350e-01	-3.0421e-01	-2.7668e-02	-9.2901e-01

Table B.2b. Parameters of Dynamic Projective Controller for CRY Model.

$$\begin{vmatrix} H & D \\ N_d & K_d \end{vmatrix} =$$

-5.6284e-01	7.5308e-01	-6.6506e+00	-1.0454e+01	1.3841e-01	-7.0794e-02
-7.5308e-01	-5.6284e-01	-1.2762e+01	-2.0077e+01	4.0370e-01	3.7376e-02
-1.5604e+00	-3.1599e+01	1.0855e+00	-1.1585e-01	9.2273e-01	3.4965e+00

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